

BAYESIAN TESTING OF EQUALITY AND ORDER- CONSTRAINED HYPOTHESES:

APPLICATIONS, METHODOLOGY & SOFTWARE

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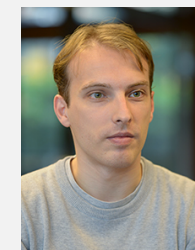
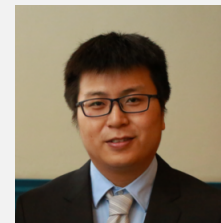
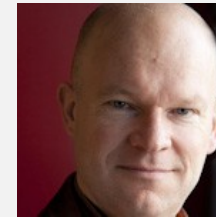


OVERVIEW

- Applications of equality and order constrained hypothesis testing
 - Linear regression in organizational psychology
 - Multilevel modeling in educational measurement
 - Relational event modeling of dynamic social networks
- Classical approaches
- Bayes factors
 - Basics, properties, behavior
- Software for Bayes factor testing of constrained hypotheses
- Applications revisited

JOINT WORK WITH...

- Herbert Hoijtink (Utrecht University);
- Roger Leenders (Tilburg University);
- Jean-Paul Fox (University of Twente);
- Eric-Jan Wagenmakers (University of Amsterdam);
- Irene Klugkist (Utrecht University);
- Gu Xin (University of Liverpool);
- Dino Dittrich (Tilburg University);
- Florian Boing-Messing (Tilburg University);
- Johan Braeken (University of Oslo);
- Stephen Wood (University of Leicester).



TESTING COMPOSITE HYPOTHESES WITH EQUALITY AND ORDER CONSTRAINTS

- The magnitude of statistical parameters are **not absolute** but **relative**.
- Effects are not only **relative to each other**, but also to the **area of the science**, and to the research method that is employed (Cohen, 1988).

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- For example, a (standardized) effect of .3 may be large when regressing **political left-right placement** on **educational level**, but small when regressing **quality-of-life** on **treatment dosage**.

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- For example, a (standardized) effect of .3 may be large when regressing **political left-right placement** on **educational level**, but small when **regressing quality-of-life** on **treatment dosage**.
- Therefore test composite hypotheses with **equality** (=) and **order constraints** (<, >) on the effects of interest.

ORDER HYPOTHESES ON RELATIVE EFFECTS

- **Workplace aggression** from inside and outside the organization.
- Employees at hospitals indicated whether they experienced any form of aggression by **managers, coworkers**, and/or **visitors**.
- The degree of **anxiety, depression**, and **job dissatisfaction** was measured for each respondent which served as dependent variables.



ORDER HYPOTHESES ON RELATIVE EFFECTS

- **Relative effects of independent variables on a dependent variable** are generally modeled using regression models.
- An example of workplace aggression on workers' well-being.

$$\text{Depression} = \beta_0 + \beta_V \times \text{Aggression}(V) + \beta_C \times \text{Aggression}(C) + \beta_M \times \text{Aggression}(M) + \text{error}$$

- with β_V = Relative effect of aggression from **visitors** on depression.
 β_C = Relative effect of aggression from **colleagues** on depression.
 β_M = Relative effect of aggression from **managers** on depression.

ORDER HYPOTHESES ON RELATIVE EFFECTS

- Hypotheses are often formulated with order constraints.
 - We expected a **positive effect** of aggression from visitors on depression, i.e., $\beta_V > 0$.
 - We expected a **positive effect** of aggression from coworkers on depression, i.e., $\beta_C > 0$.
 - We expected a **positive effect** of aggression from managers on depression, i.e., $\beta_M > 0$.
 - Based on the **hierarchy in the organization** we expected that the relative effect of aggression on depression is largest for managers, followed by coworkers, followed by visitors: $\beta_M > \beta_C > \beta_V$.
- See Braeken et al. (2014).

ORDER HYPOTHESES ON RELATIVE EFFECTS

- We can combine these order constraints into a single 'order hypothesis'.
- "All sources of workplace aggression have a positive effect on depression and the effect is largest for managers, followed by coworkers, followed by visitors":

Power differential hypothesis: $H_1: \beta_M > \beta_C > \beta_V > 0$

versus

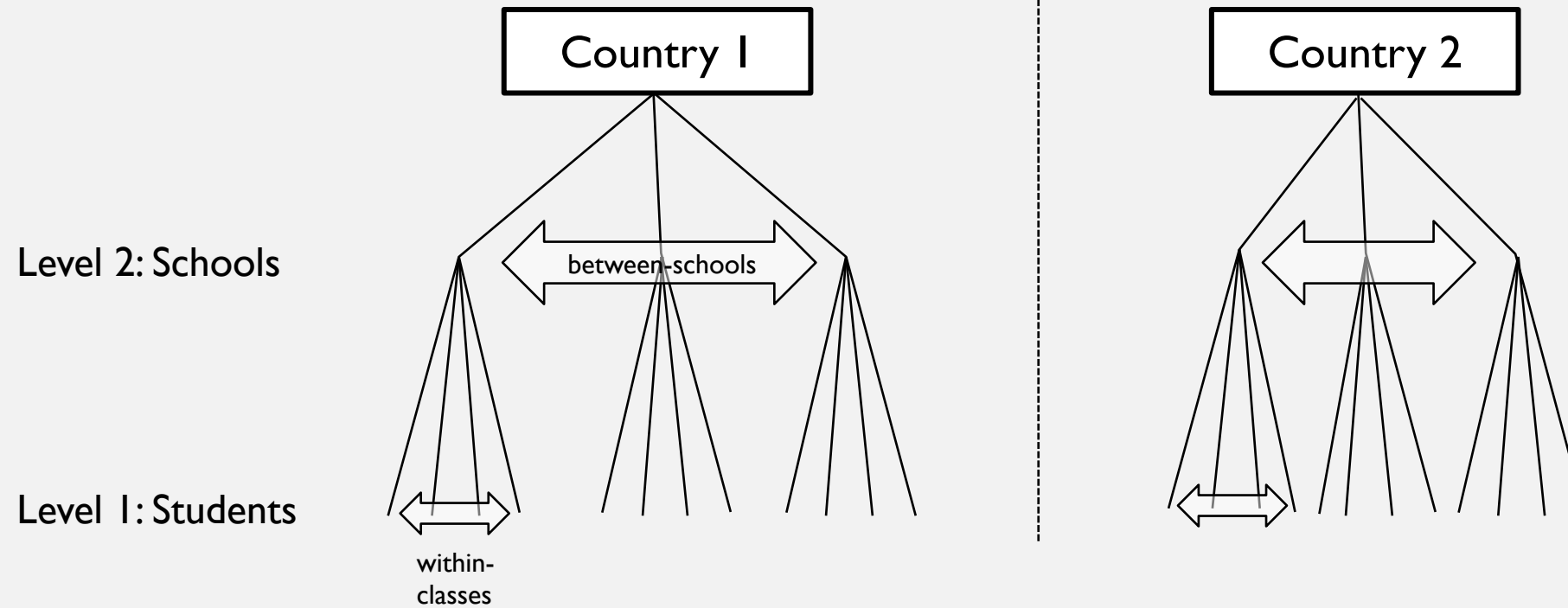
Complement hypothesis: H_2 : "not H_1 ".

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS

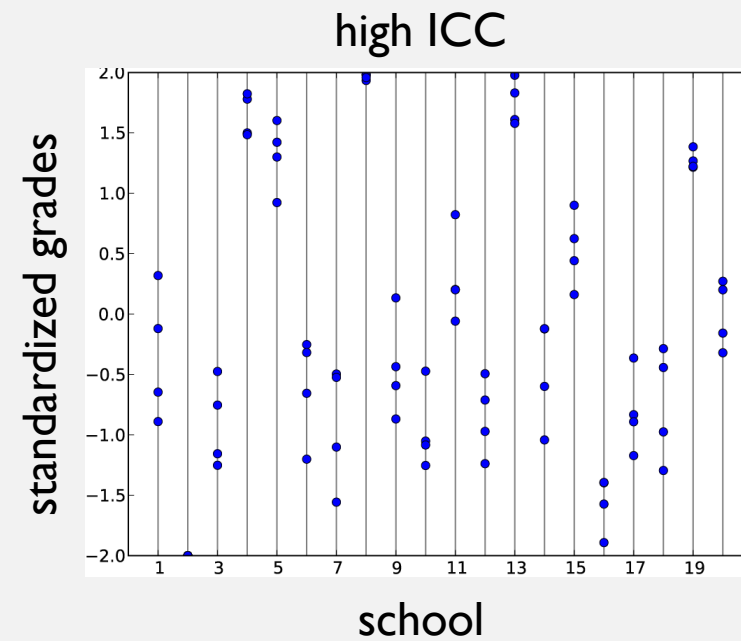
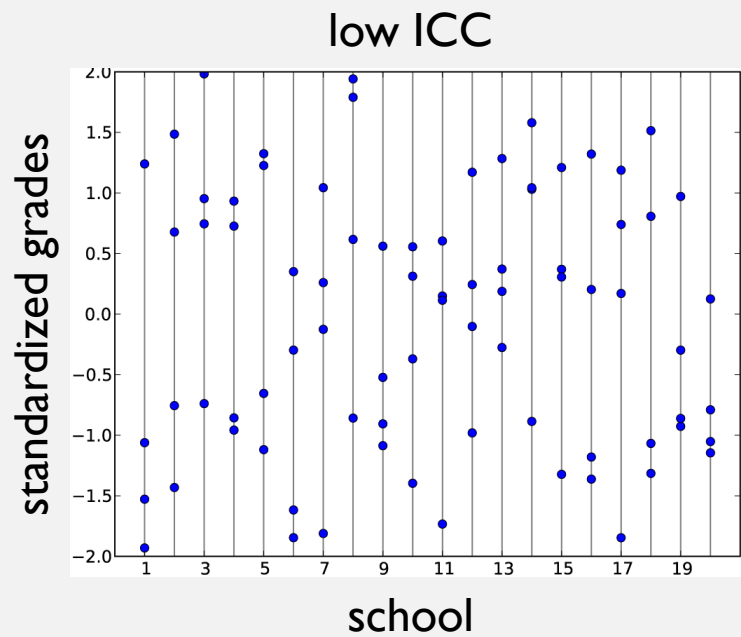
- The Trends in International Mathematics and Science Study (**TIMSS**) measures the performances of fourth and eight graders in more than 50 participating countries around the world (www.iea.nl/timss).
- Interest is not only in testing the average grades across countries, but also in testing the **variability of school performance within countries**.
- The relative variability between schools is quantified by **country specific intraclass correlations**.



ORDER HYPOTHESES ON INTRACLASS CORRELATIONS



ORDER HYPOTHESES ON INTRACLASS CORRELATIONS



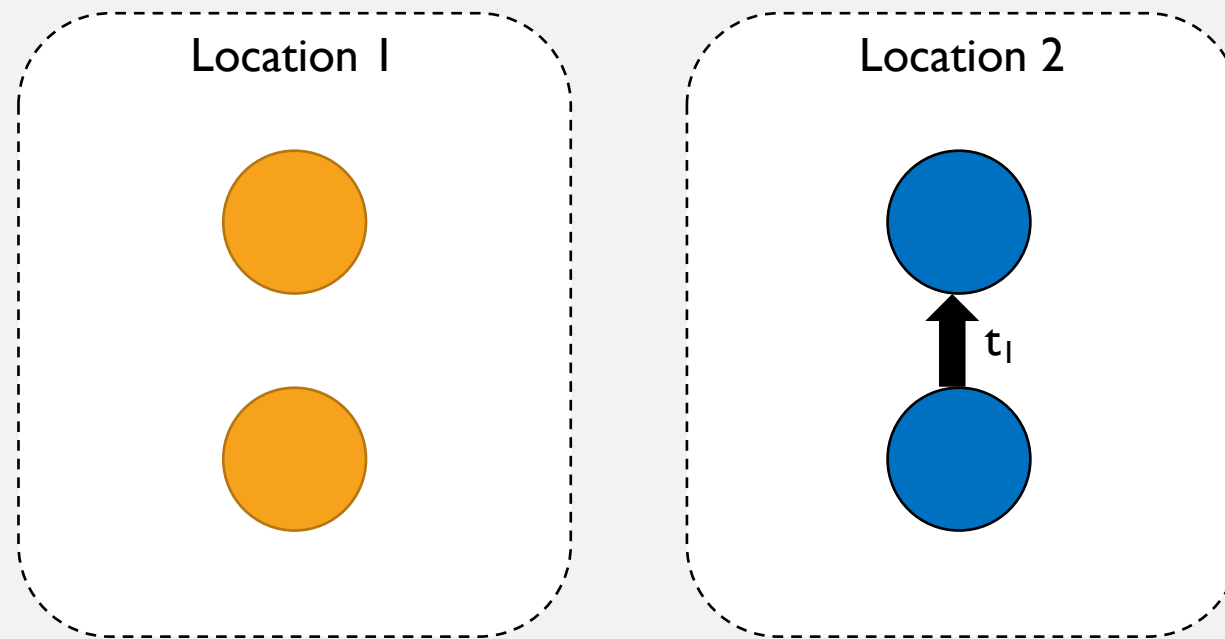
ORDER HYPOTHESES ON INTRACLASS CORRELATIONS

- Focus on The Netherlands, Croatia, Germany, and Denmark.
- **Null hypothesis:** Equal intraclass correlations over countries:
 - $H_0: \rho_{NL} = \rho_{CR} = \rho_{GER} = \rho_{DEN}$.
- **Order hypothesis:** Previous research suggest that between schools variability is largest for Danish schools, followed by German schools, followed by Croatian schools, followed by Dutch Schools:
 - $H_1: \rho_{NL} < \rho_{CR} < \rho_{GER} < \rho_{DEN}$.
- **Complement hypothesis:**
 - H_2 : neither H_0 , nor H_1 .

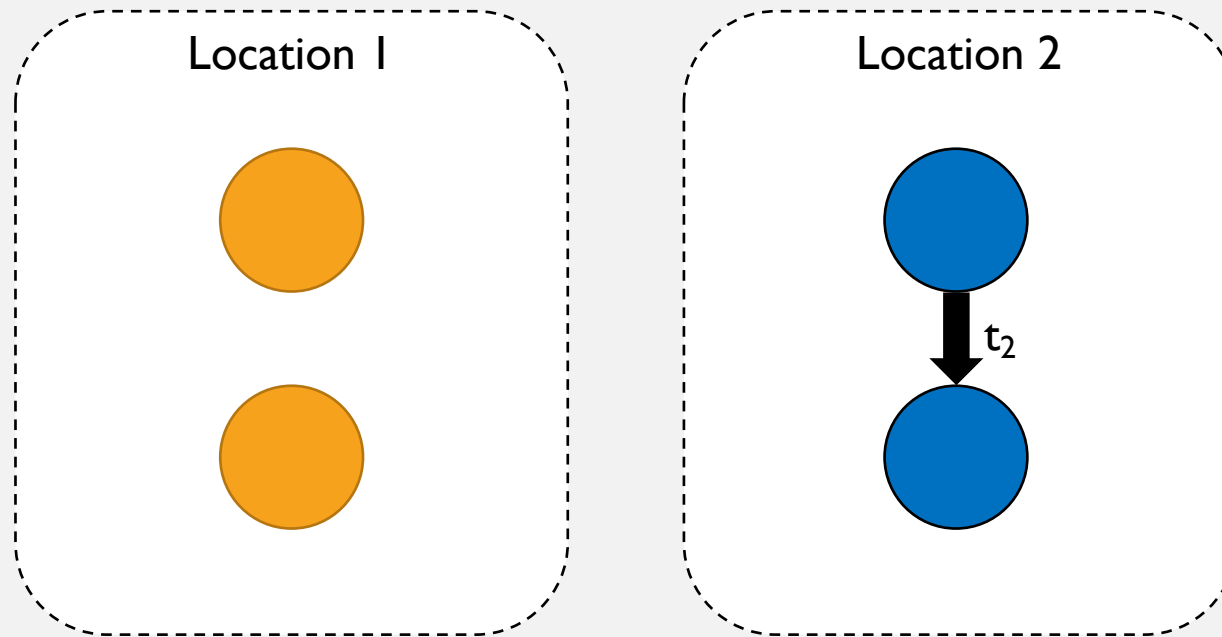
ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS

- A **relational event history data** was collected of email messages between colleagues containing information about innovative ideas in a large consultancy firm.
- The data contain information about **who** send a message **to whom** and **when** in the year 2010.
- Interest was in the **relative importance** of drivers of these social interactions between colleagues and how this changes over time.
- The relational event model of Butts (2008) was employed.

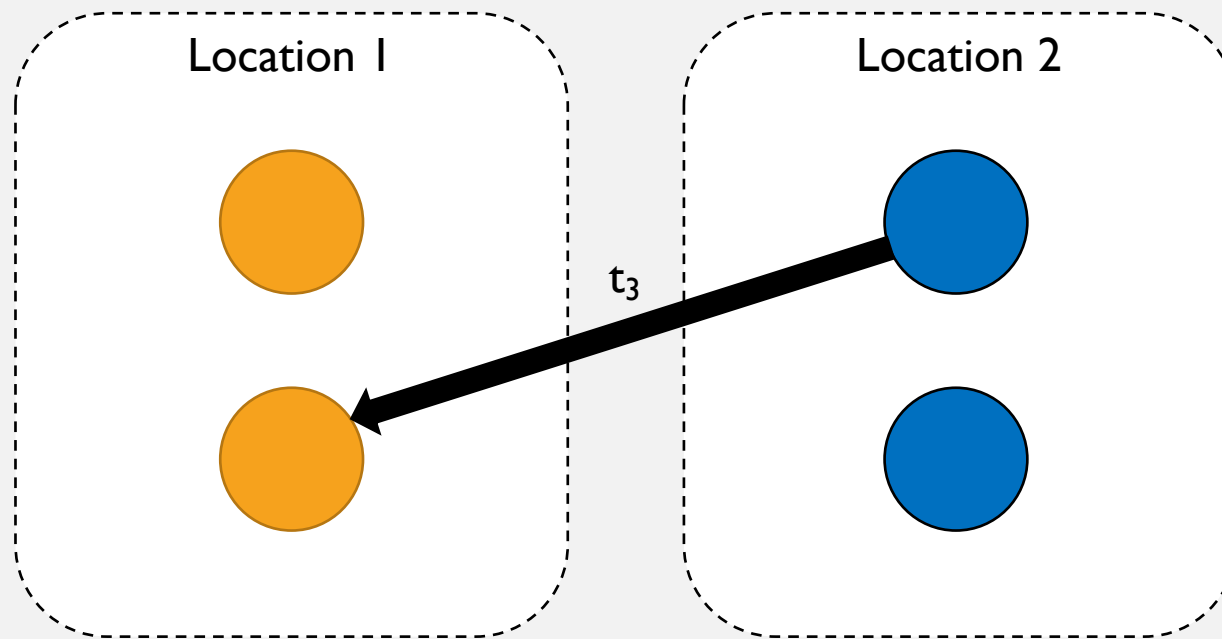
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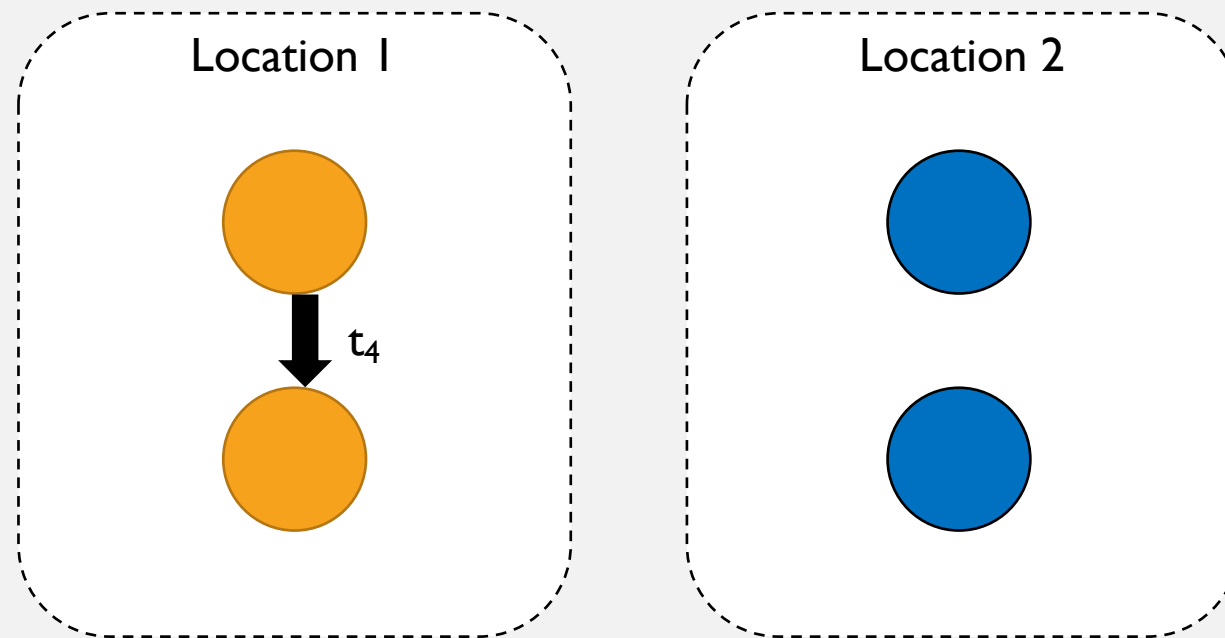
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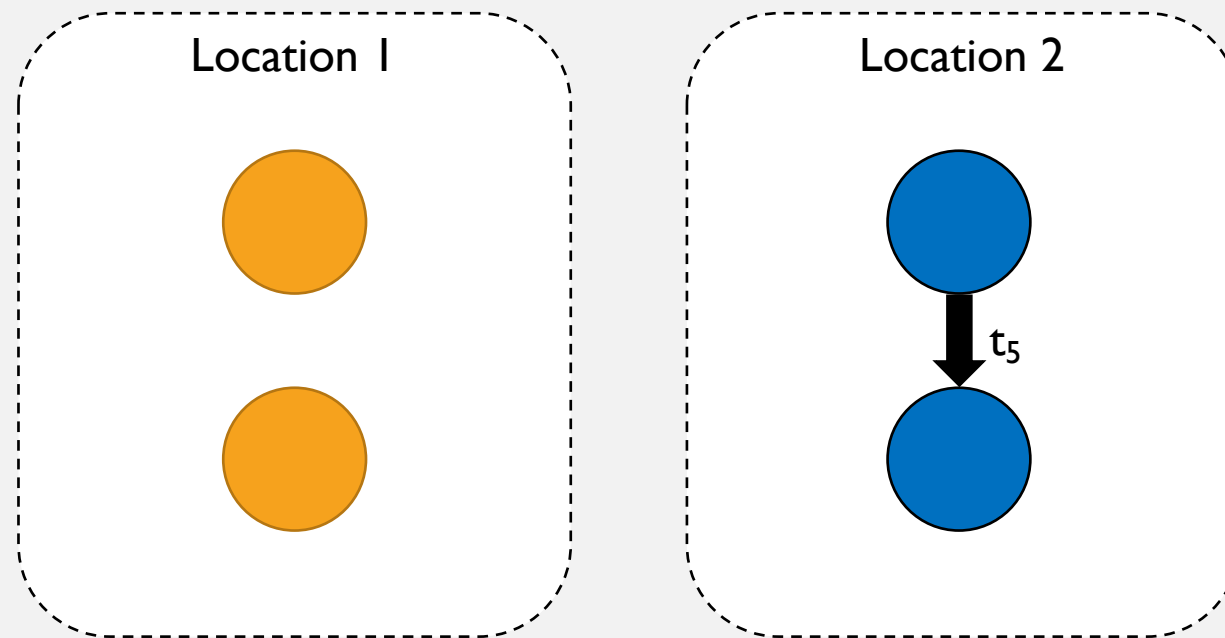
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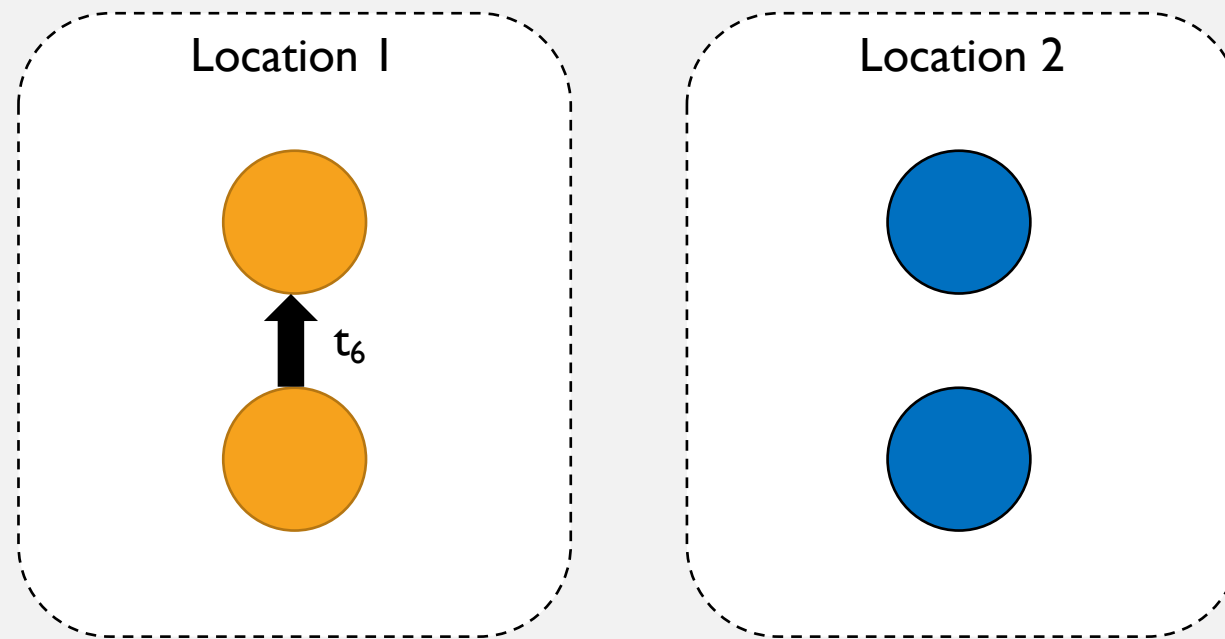
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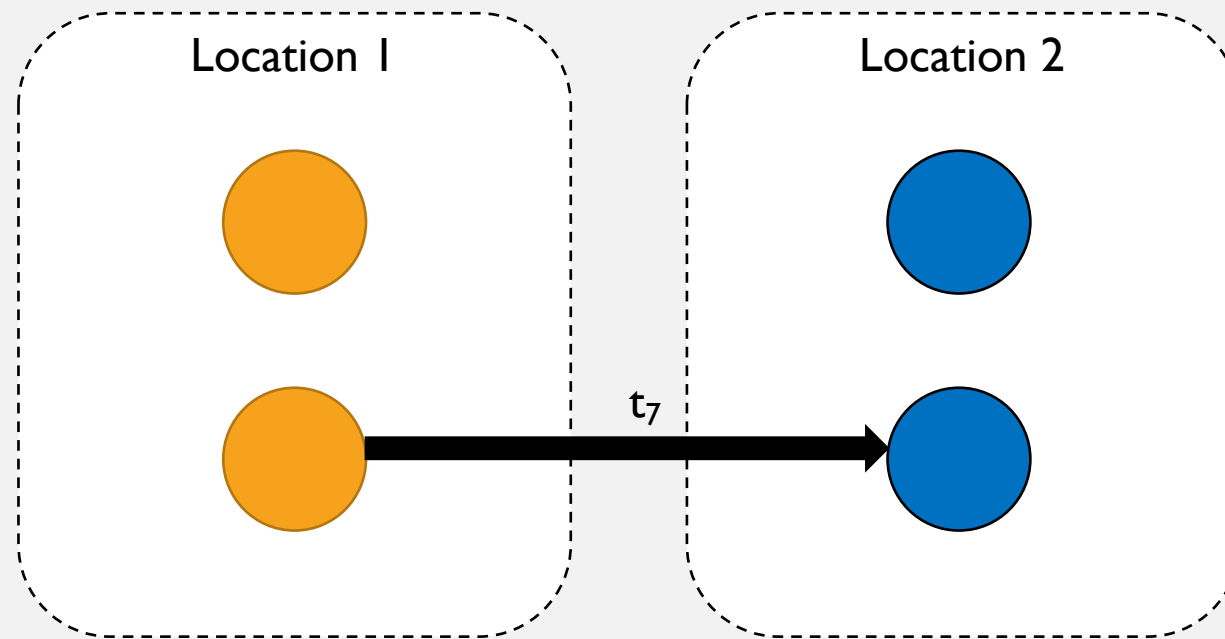
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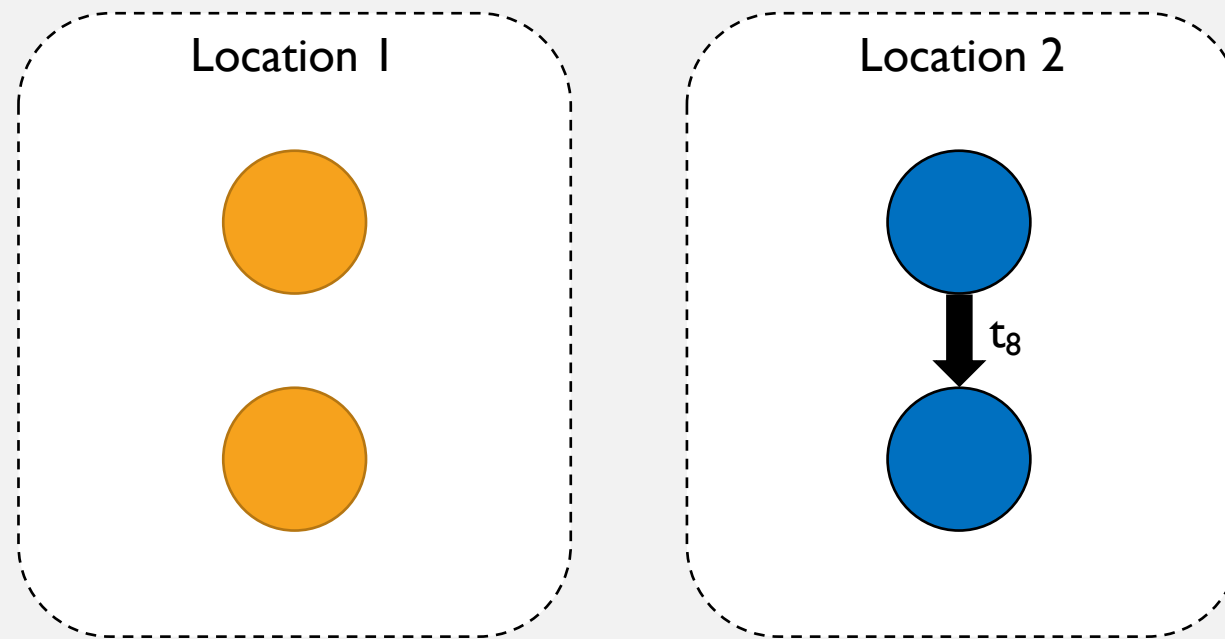
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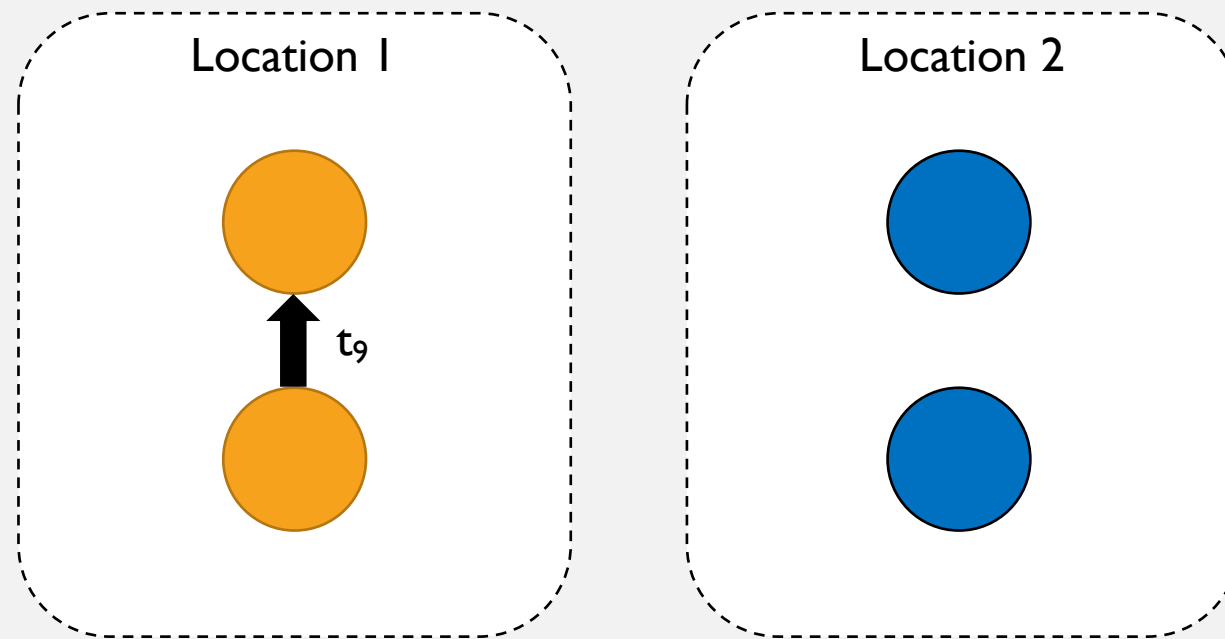
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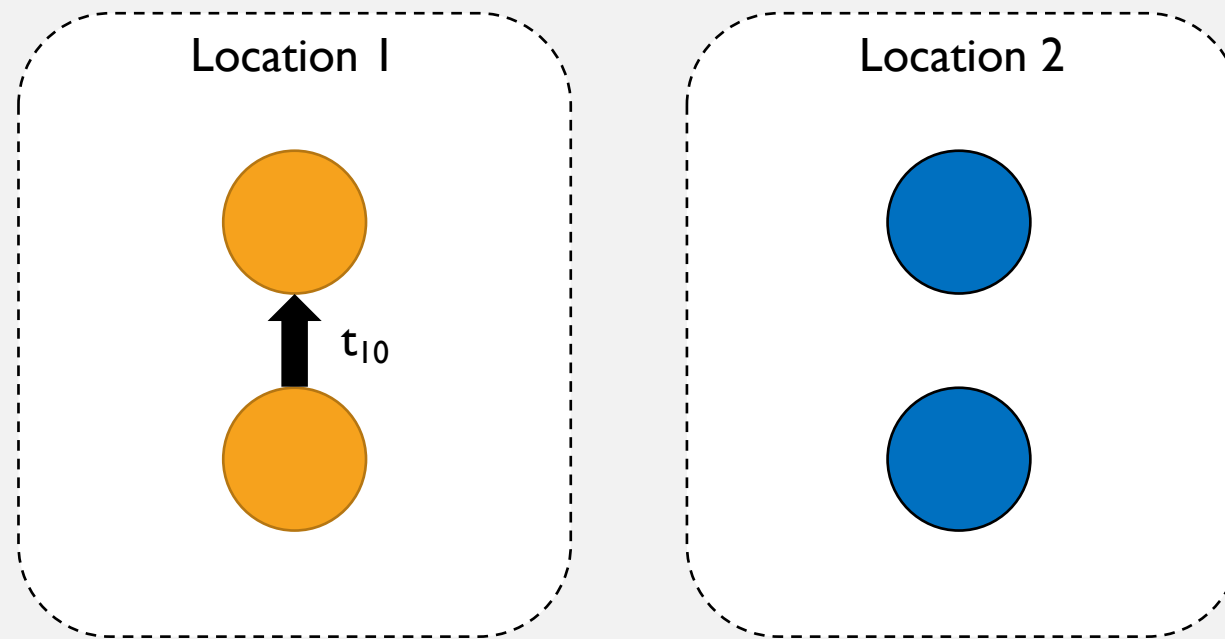
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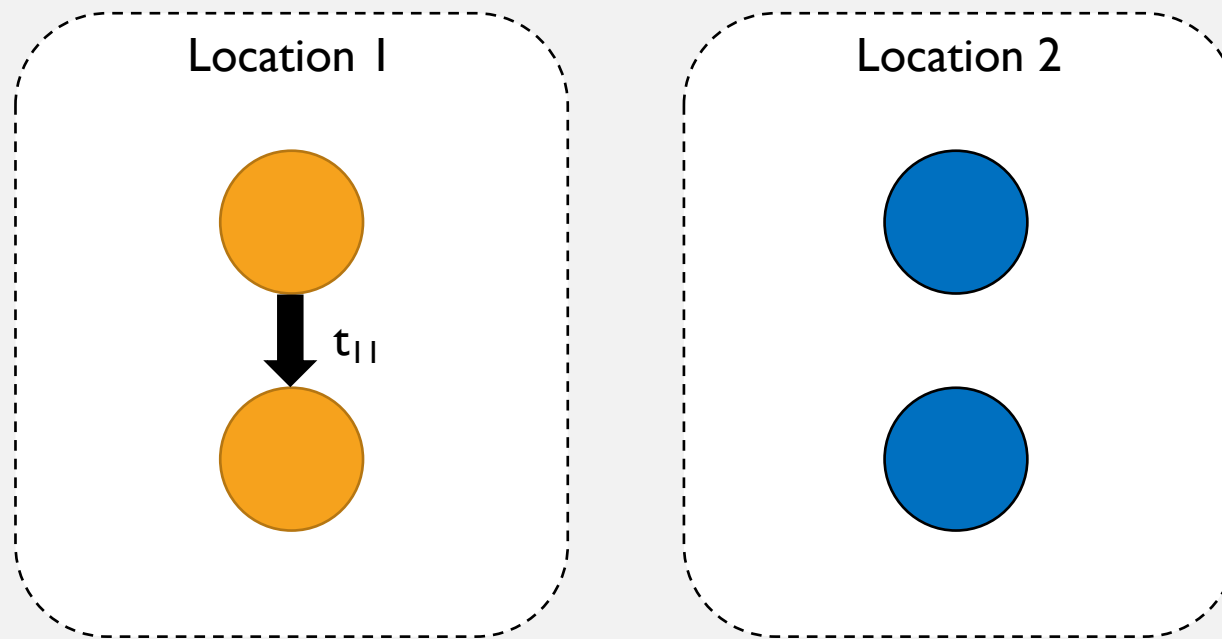
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ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS

- It is often suggested that information-sharing occurs sooner and at a higher rate among colleagues who they feel related to – this is often attributed to **identity**.
- Thus, having the same position may have a positive and strong effect on information sharing, while working in the same building, or being of the same gender may also have a positive effect but to a lesser extent.

- $H_1: \beta_{\text{position}} > \beta_{\text{building}} > \beta_{\text{gender}} > 0$

- Alternatively, only being in the same location may have an effect.

- $H_2: \beta_{\text{position}} > \beta_{\text{building}} = \beta_{\text{gender}} = 0$

- $H_3: \text{neither } H_1, \text{ nor } H_2.$

WHY TEST ORDER HYPOTHESES?

- Why test order hypotheses instead of the classical null and alternative hypotheses?
 - We test the magnitude of the **effects relative to the scientific field and study**.
 - We get a **direct answer** whether our theory or expectations (with order constraints) are supported by the data.
 - We test with more statistical **power**.
 - Compare the two-tailed test ($\beta \neq 0$) with the one-tailed test ($\beta > 0$).
 - More power implies that we are more likely to draw the correct conclusions.

HOW CAN WE TEST ORDER HYPOTHESES?

- **Significance tests using p-value...**
 - ...are only available for testing **specific types of order hypotheses** for 'standard' models (Silvapulle & Sen, 2004);

$H_0: \beta_1 = \beta_2 = \beta_3$ against $H_1: \beta_1 > \beta_2 > \beta_3$ **YES**

$H_0: \beta_1 > \beta_2 > \beta_3$ against $H_1: \beta_1, \beta_2, \beta_3$ **YES**

$H_0: \beta_1 = \beta_2 > \beta_3$ against $H_1: \beta_1, \beta_2, \beta_3$ **NO**

$H_0: \beta_1 > \beta_2 > \beta_3$ against $H_1: \beta_1 < \beta_2 < \beta_3$ **NO**

HOW CAN WE TEST ORDER HYPOTHESES?

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$\beta_1 \neq 0$ (sign.), $\beta_2 = 0$ (not sign.), $\beta_1 = \beta_2$ (not sign.).

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 - ...are **inconsistent** when the null is true;

$$H_0: \beta_1 = \beta_2 = \beta_3 \text{ against } H_1: \beta_1 > \beta_2 > \beta_3$$

For extremely large samples there is still a probability of α to incorrectly reject a true H_0 .

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 - ...are **inconsistent** when the null is true;
 - ...are not designed for simultaneously testing **multiple (more than 2) hypotheses**.

$$H_0: \beta_1 > \beta_2 > \beta_3 \quad \text{vs} \quad H_1: \beta_1 < \beta_2 < \beta_3 \quad \text{vs} \quad H_2: \text{neither } H_0, \text{ nor } H_1.$$

HOW CAN WE TEST ORDER HYPOTHESES?

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 - ...are **inconsistent** when the null is true;
 - ...are not designed for simultaneously testing **multiple (more than 2) hypotheses**.
 - ...depend on the **sampling plan**.

HOW CAN WE TEST ORDER HYPOTHESES?

- **(Classical) information criteria**, such as the AIC or BIC, are not designed for testing models with order hypotheses on the parameters (Mulder et al., 2009).

- $$\text{AIC} = 2d - 2\log(\hat{L})$$

Model complexity
(number of free parameters)

Model fit
(maximized likelihood)

- How many **free parameters** d does $H_1: \beta_M > \beta_C > \beta_V > 0$ have?

BAYES FACTORS

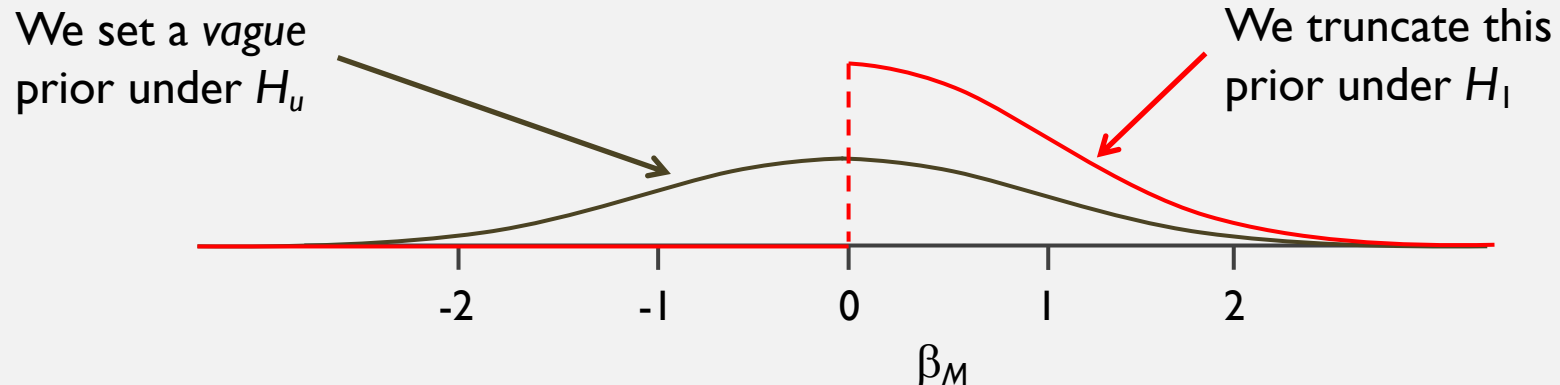
- The **Bayes factor** (Jeffreys, 1961) is defined as the ratio of the marginal likelihoods

$$BF(H_1, H_2) = \frac{\text{marginal likelihood}(H_1)}{\text{marginal likelihood}(H_2)}$$

- The **marginal likelihood** of H_1 quantifies how plausible the data were generated under hypothesis H_1 .
- For example, a Bayes factor of $BF(H_1, H_2) = 10$ implies that it was 10 times more plausible that the data were generated under H_1 than under H_2 .
- Marginal likelihoods can be **difficult to compute**.
- Bayes factors can be **sensitive to the prior**.

BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

- When testing order hypotheses we can use the *encompassing prior* approach to set priors under the hypotheses (e.g., Klugkist et al., 2005).
- Consider $H_1: \beta_M > 0$ versus $H_u: \beta_M$ unconstrained,



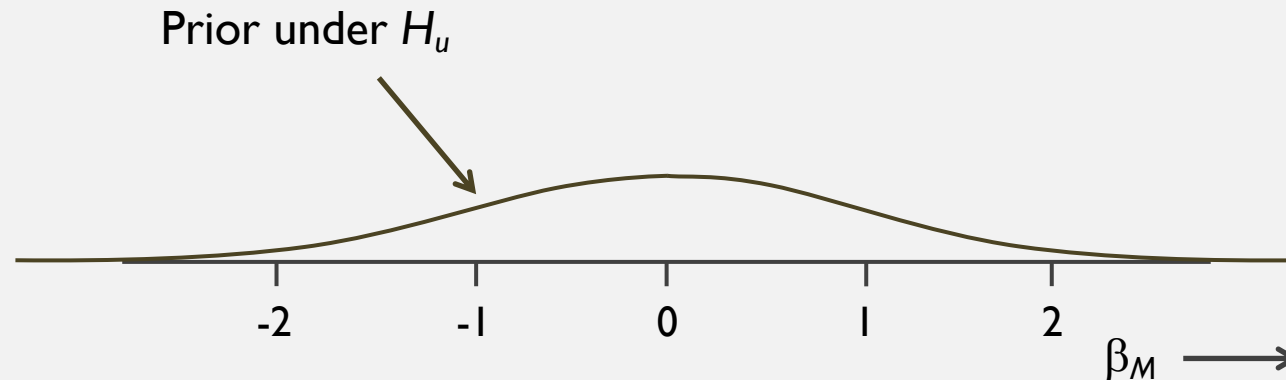
BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

- The Bayes factor of an order hypothesis H_1 against the unconstrained hypothesis H_u is then equal to

$$\begin{aligned} \text{BF}(H_1, H_u) &= \frac{\text{posterior probability that the constraints of } H_1 \text{ hold under } H_u}{\text{prior probability that the constraints of } H_1 \text{ hold under } H_u} \\ &= \frac{\text{Relative fit of } H_1}{\text{Relative complexity of } H_1} \end{aligned}$$

BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

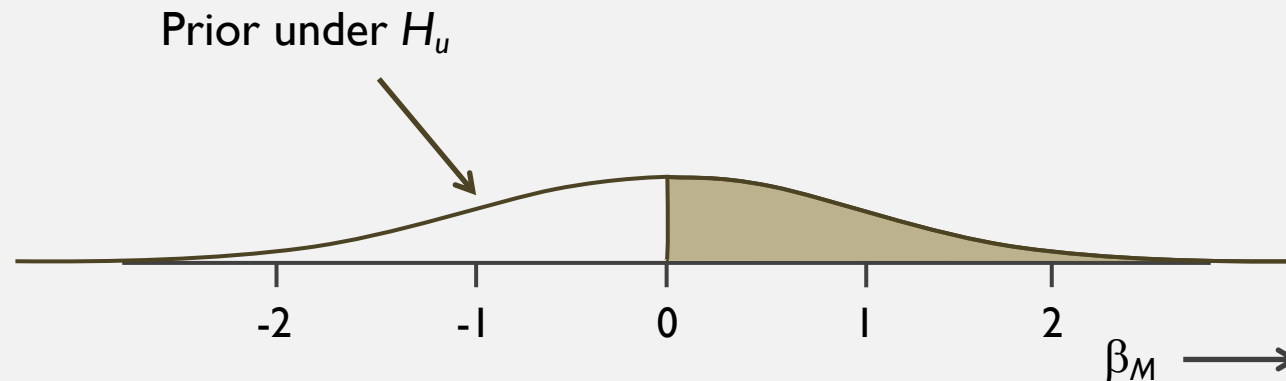
- For example, $H_1: \beta_M > 0$ versus $H_u: \beta_M$ unconstrained,



- $$BF(H_1, H_u) = \frac{\text{posterior probability that } \beta_M > 0}{\text{prior probability that } \beta_M > 0}$$

BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

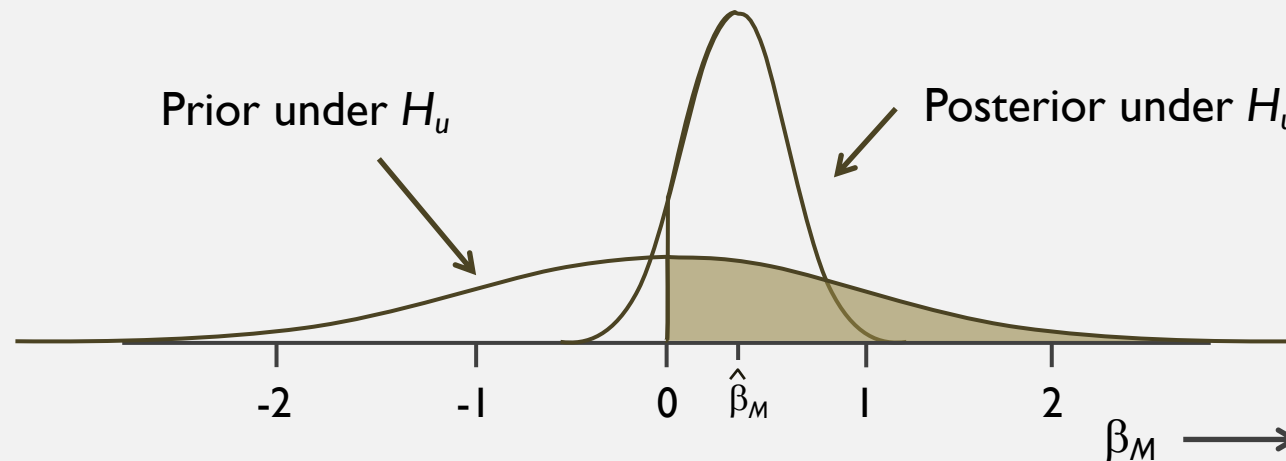
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- $$BF(H_1, H_u) = \frac{\text{posterior probability that } \beta_M > 0}{\text{prior probability that } \beta_M > 0} = \frac{\quad}{0.5}$$

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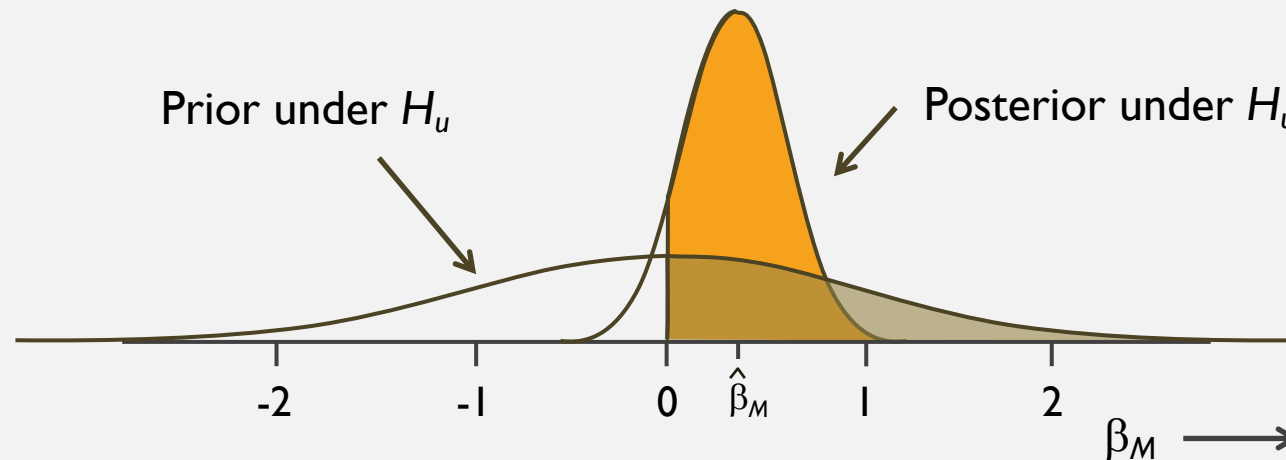
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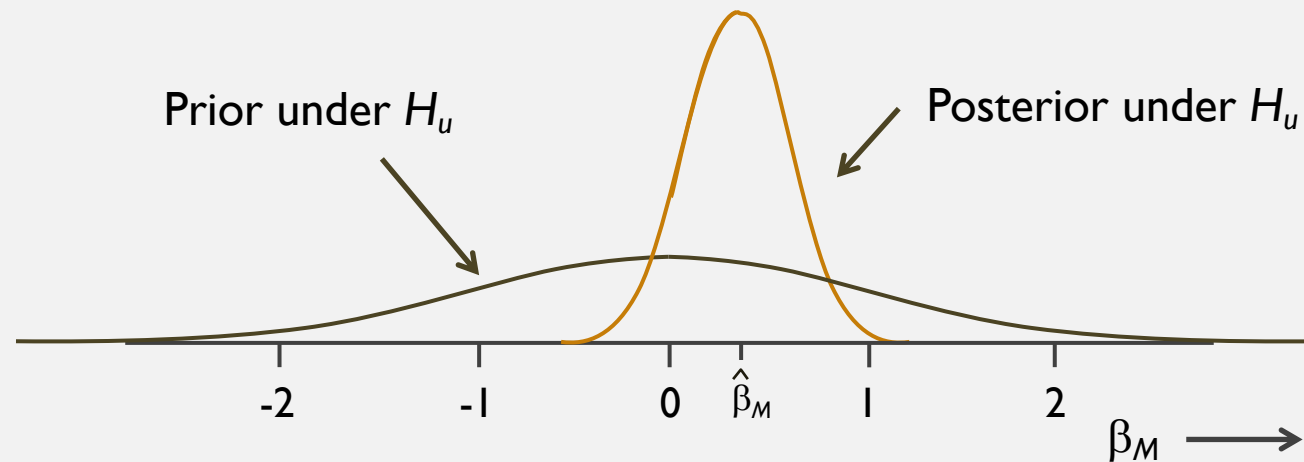
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- $$BF(H_1, H_u) = \frac{\text{posterior probability that } \beta_M > 0}{\text{prior probability that } \beta_M > 0} = \frac{0.9}{0.5} = 1.8$$

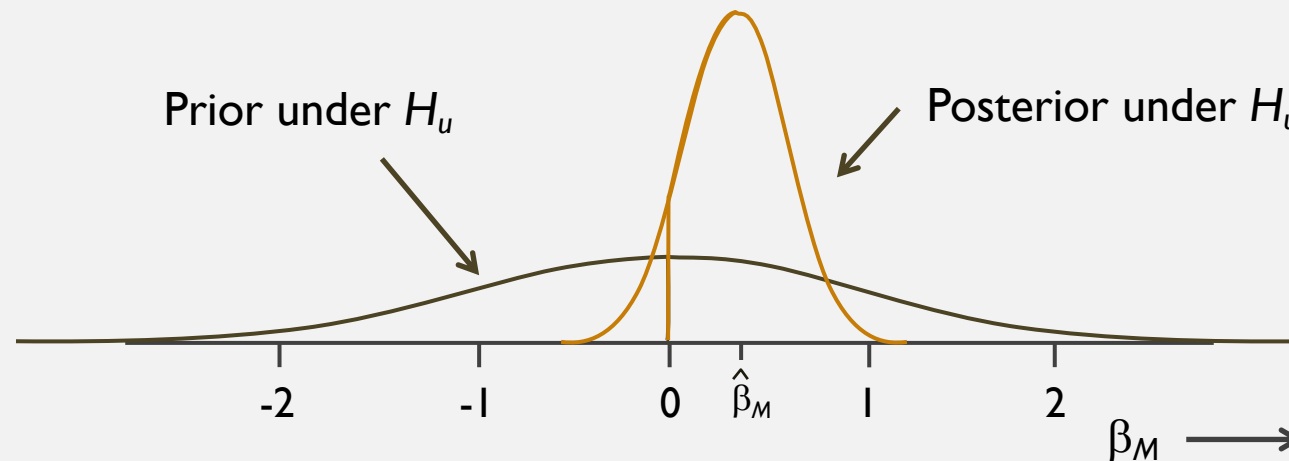
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BAYES FACTORS FOR TESTING A PRECISE HYPOTHESIS

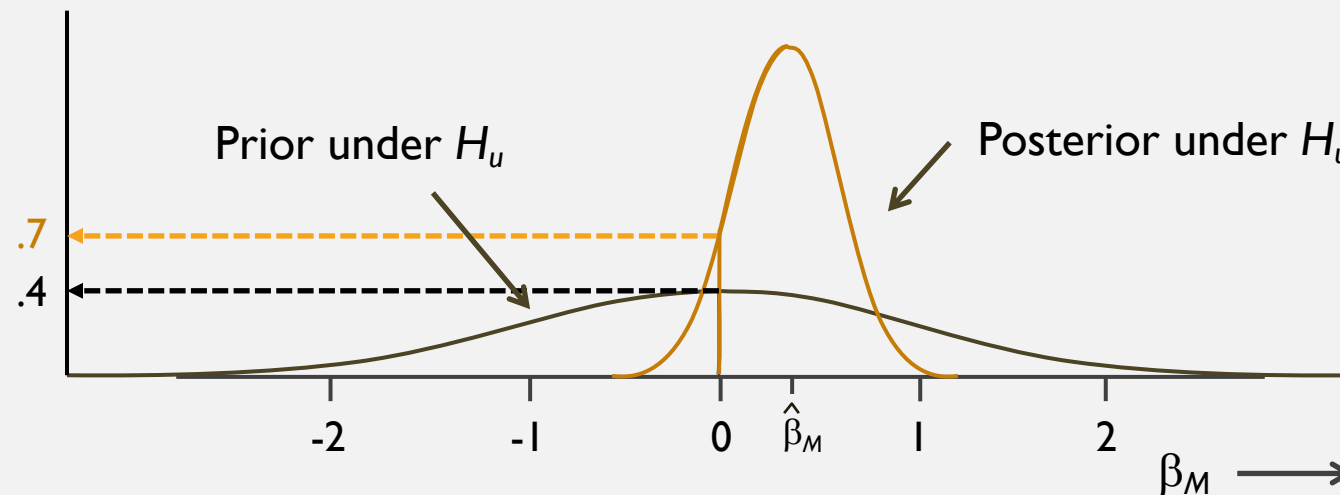
- For example, $H_0: \beta_M = 0$ versus $H_1: \beta_M \neq 0$,



- $$BF(H_0, H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0}$$

BAYES FACTORS FOR TESTING A PRECISE HYPOTHESIS

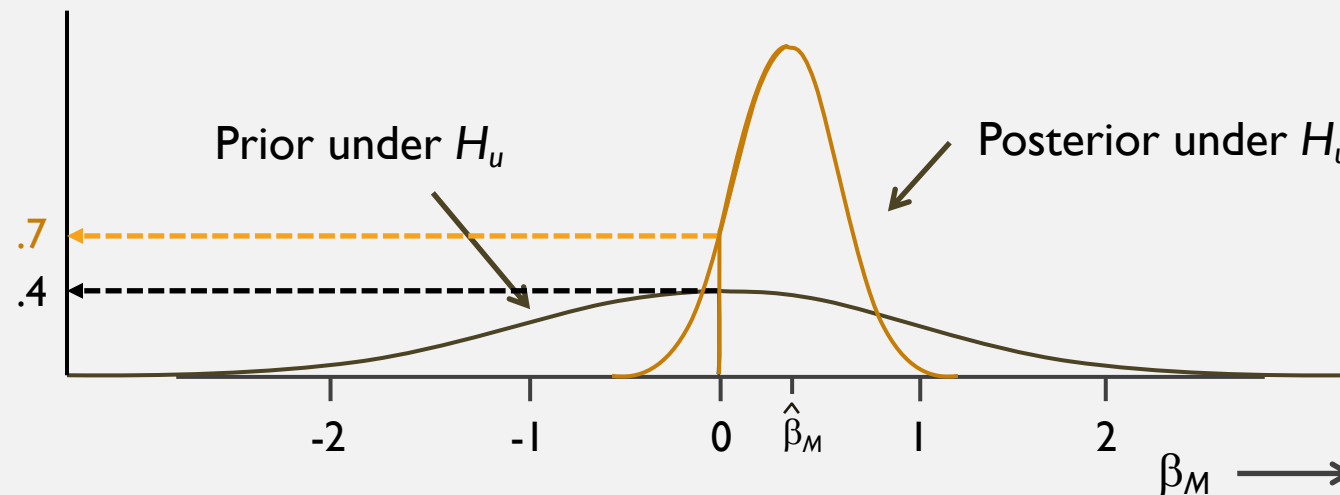
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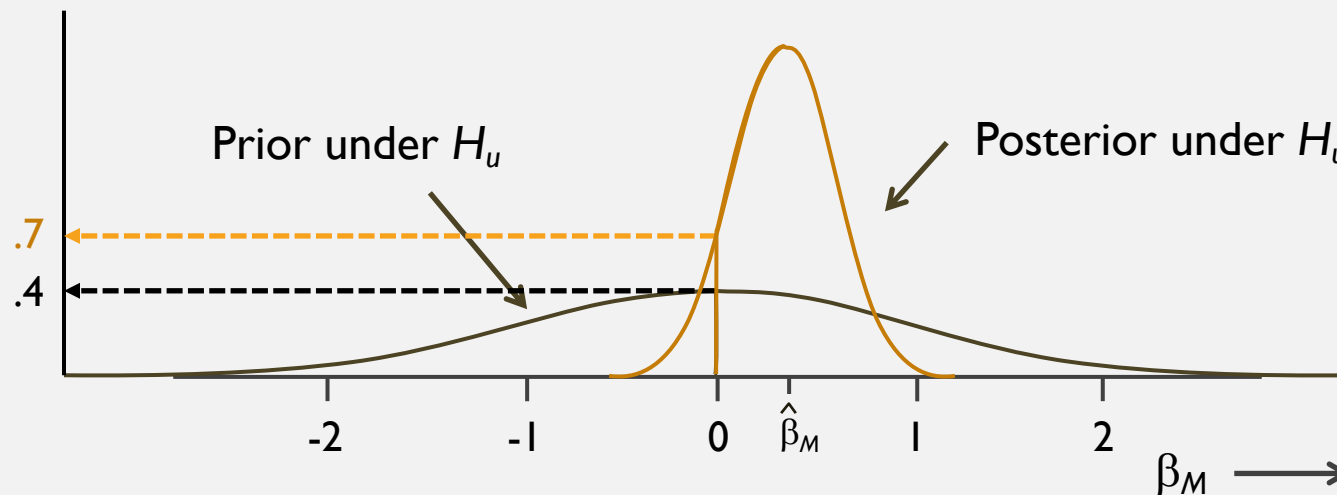
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- $$BF(H_0, H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0} = \frac{.7}{.4} = 1.75$$

BAYES FACTORS FOR TESTING A PRECISE HYPOTHESIS

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The Savage-Dickey density ratio of the Bayes factor holds for a specific choice of the priors.

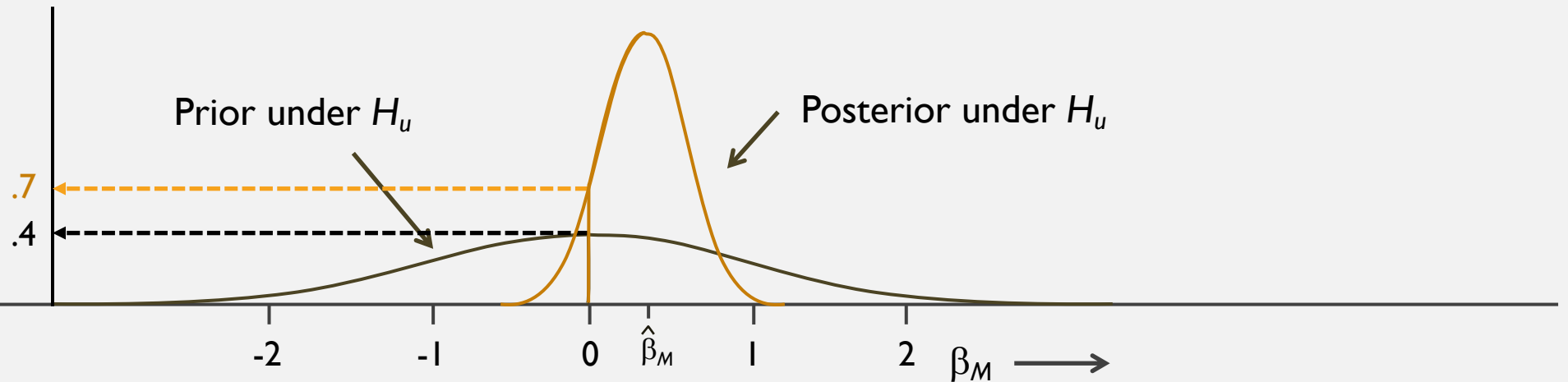
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SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- **Bartlett's paradox:** The evidence for a null hypothesis $H_0: \beta_M = 0$ against $H_1: \beta_M \neq 0$ can be made arbitrarily large by setting the prior variance large enough.

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

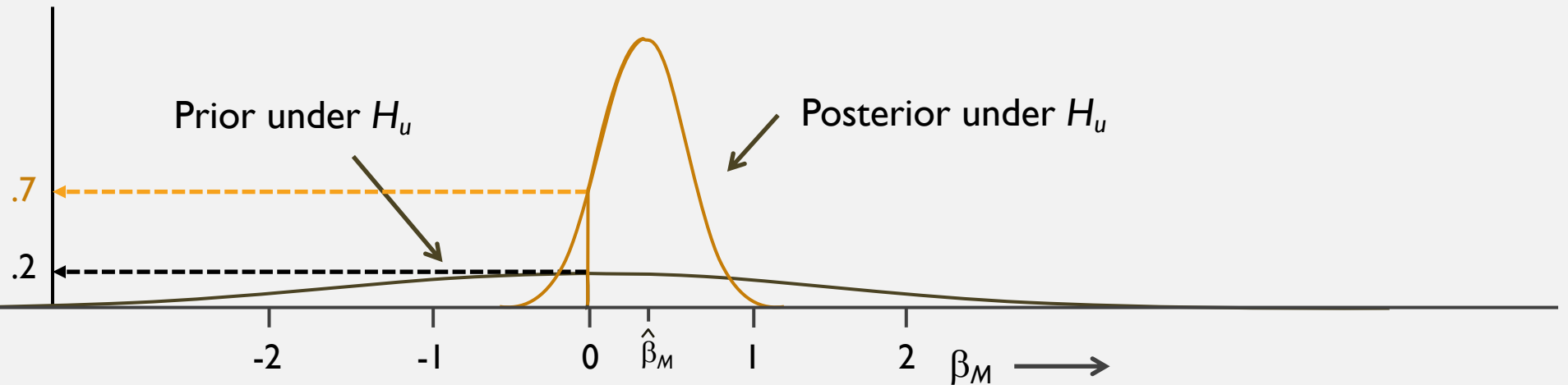
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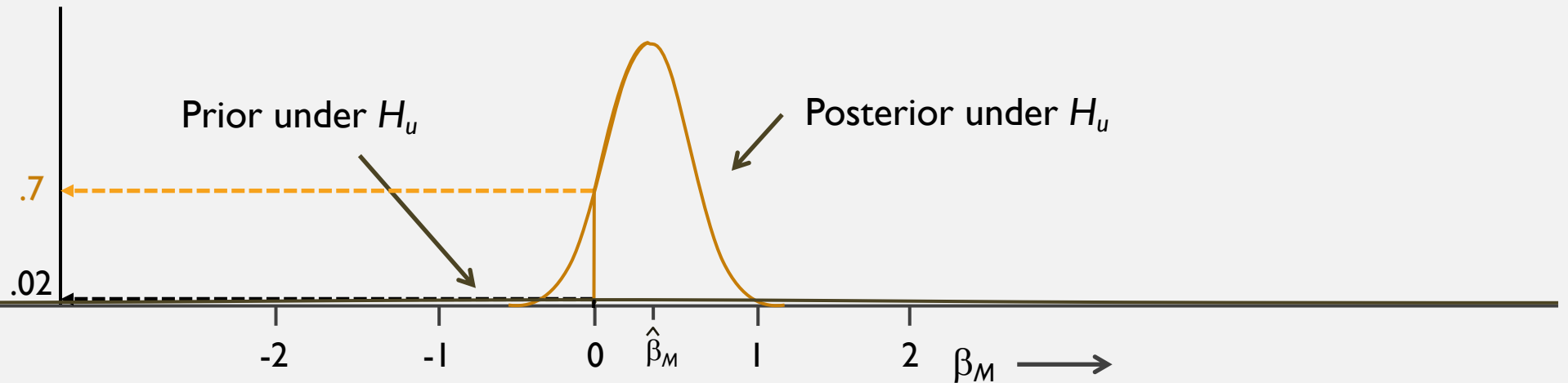
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$$\text{BF}(H_0, H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0} = \frac{.7}{.2} = 3.50$$

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

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$$BF(H_0, H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0} = \frac{.7}{.02} = 35$$

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

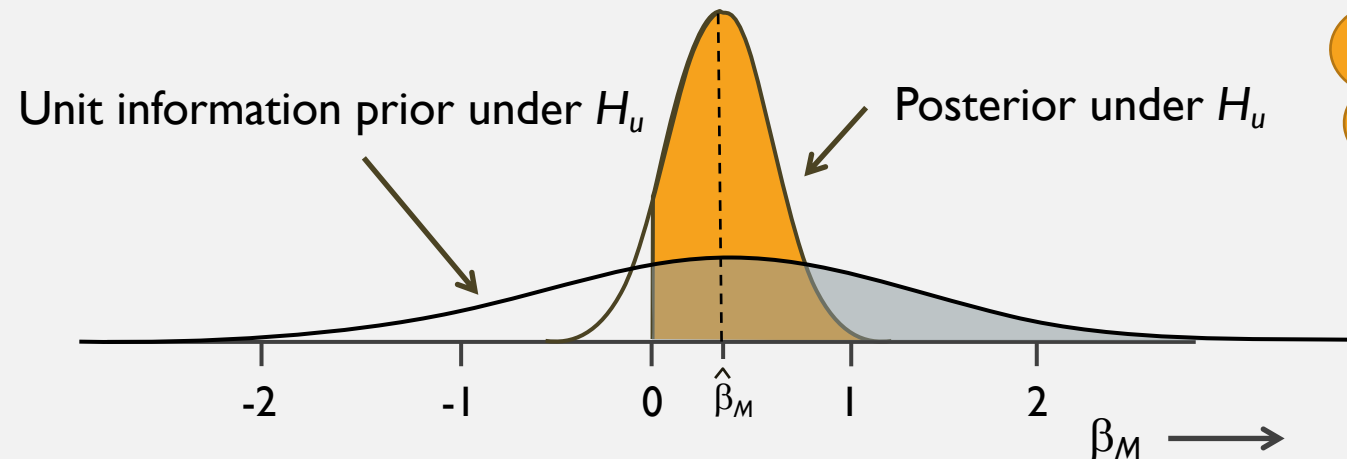
- **Bartlett's paradox:** The evidence for a null hypothesis $H_0: \beta_M = 0$ against $H_1: \beta_M \neq 0$ can be made arbitrarily large by setting the prior variance large enough.
- If clear prior information is absent use a default prior that contains the **information of a minimal experiment**.

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- **Bartlett's paradox:** The evidence for a null hypothesis $H_0: \beta_M = 0$ against $H_1: \beta_M \neq 0$ can be made arbitrarily large by setting the prior variance large enough.
- If clear prior information is absent use a default prior that contains the **information of a minimal experiment**.
- The **BIC** is a Bayes factor approximation based on a **unit information prior** (Schwarz, 1978; Raftery, 1995).

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

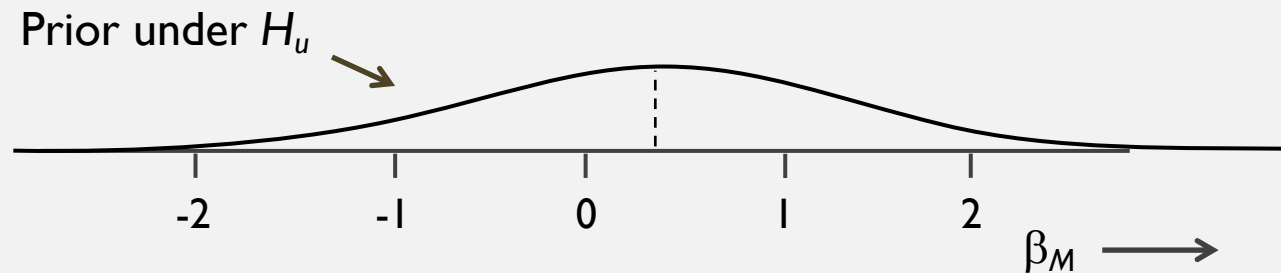
- The **unit information prior** as centered at the ML estimate.



A priori, a negative effect is not equally likely as a positive effect.

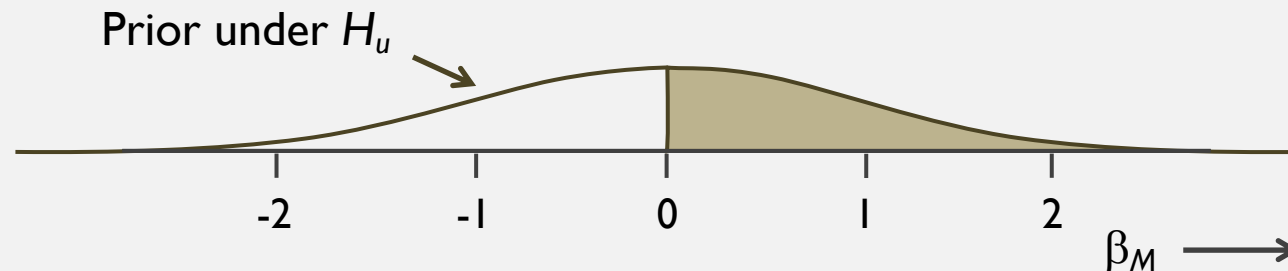
SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- A default prior should be **centered at the null value**. Then



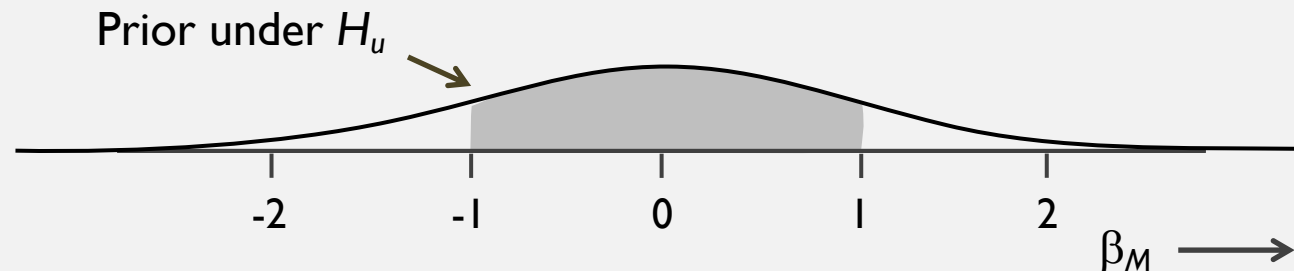
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SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- A default prior should be **centered at the null value**. Then
 - **negative effects** are equally likely as **positive effects**;
 - **small effects** are more likely a priori than **large effects**.



POSTERIOR ODDS OF HYPOTHESES

- The Bayes factors can be used to update the **prior odds** of two hypotheses to obtain the **posterior odds** between the two hypotheses.

$$\frac{P(H_1 | \text{data})}{P(H_2 | \text{data})} = \text{BF}(H_1, H_2) \times \frac{P(H_1)}{P(H_2)}$$

↑ ↑ ↑

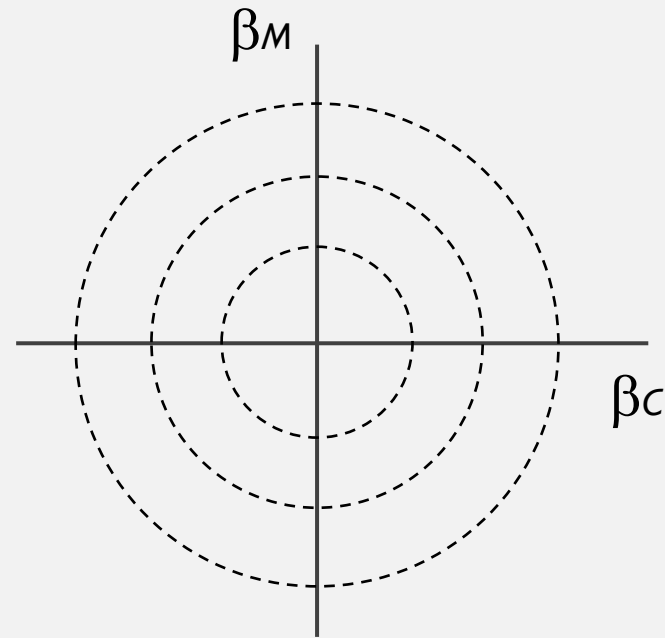
posterior odds Bayes factor prior odds

BAYES FACTORS FOR TESTING ORDER HYPOTHESES

- Example of multiple hypothesis test
 - $H_1: \beta_M > 0, \beta_C > 0$ (both effects are positive)
 - $H_2: \beta_M > \beta_C$ (effect of managers is larger than effect of coworkers)
 - $H_3: \beta_M > \beta_C > 0$ (combination of H_1 and H_2)
 - $H_u: \beta_M, \beta_C$ (no specific expectation on the effects)

BAYES FACTORS FOR TESTING ORDER HYPOTHESES

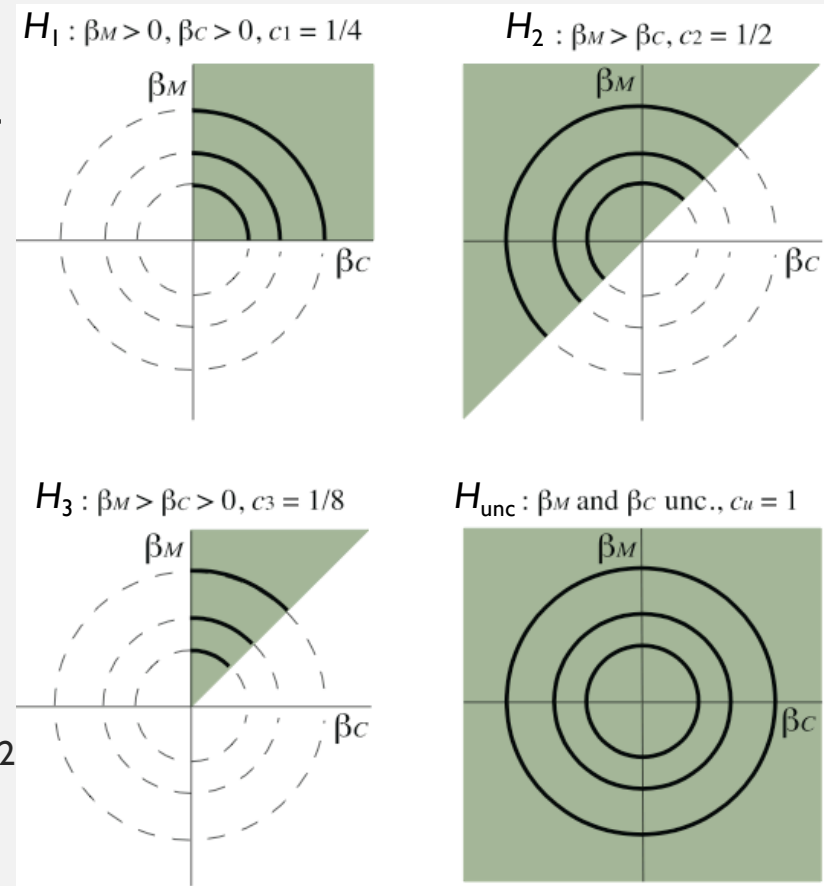
- Example of multiple hypothesis test
 - $H_1: \beta_M > 0, \beta_C > 0$
 - $H_2: \beta_M > \beta_C$
 - $H_3: \beta_M > \beta_C > 0$
 - $H_u: \beta_M, \beta_C$



A vague prior for β_C and β_M under H_u .

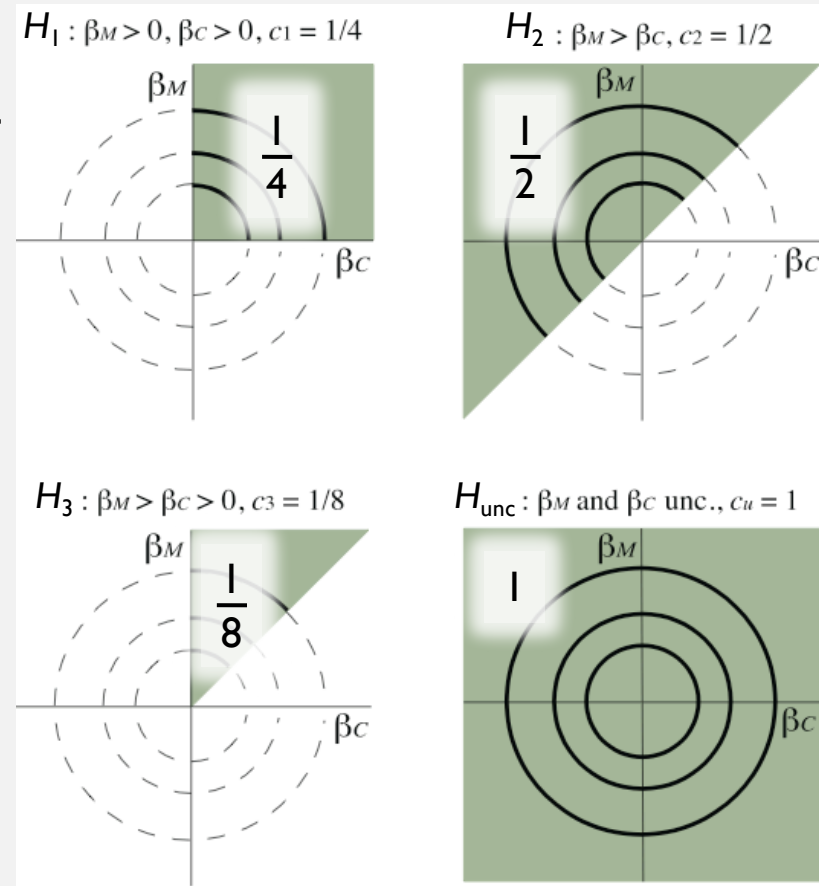
RELATIVE COMPLEXITY OF ORDER HYPOTHESES

- Example of multiple hypothesis test
 - $H_1: \beta_M > 0, \beta_C > 0$
 - $H_2: \beta_M > \beta_C$
 - $H_3: \beta_M > \beta_C > 0$
 - $H_u: \beta_M$ and β_C
- So H_u is **most complex (largest parameter space)**, followed by H_2 , H_1 , and H_3 (smallest parameter space).



RELATIVE COMPLEXITY OF ORDER HYPOTHESES

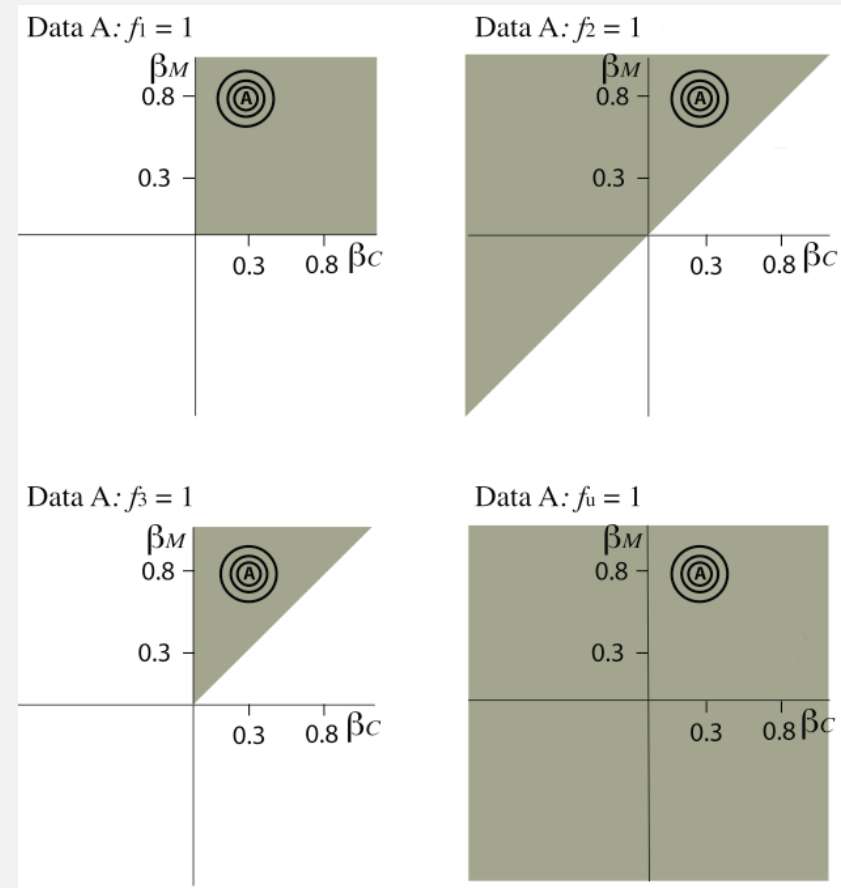
- Example of multiple hypothesis test
 - $H_1: \beta_M > 0, \beta_C > 0$
 - $H_2: \beta_M > \beta_C$
 - $H_3: \beta_M > \beta_C > 0$
 - $H_u: \beta_M$ and β_C
- **Relative complexity** (Mulder et al., 2010).



RELATIVE COMPLEXITY OF ORDER HYPOTHESES

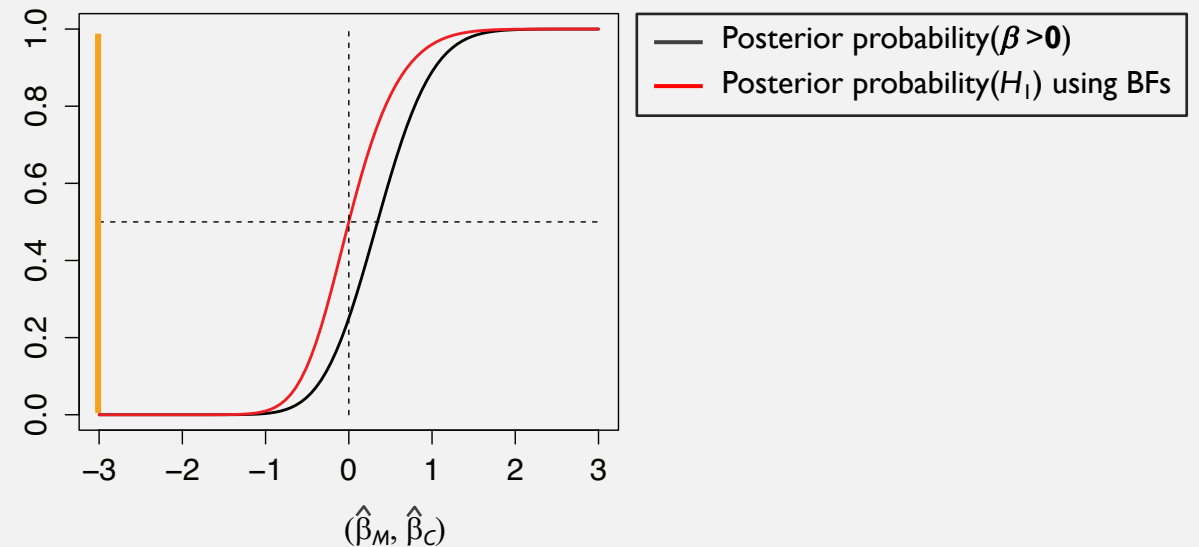
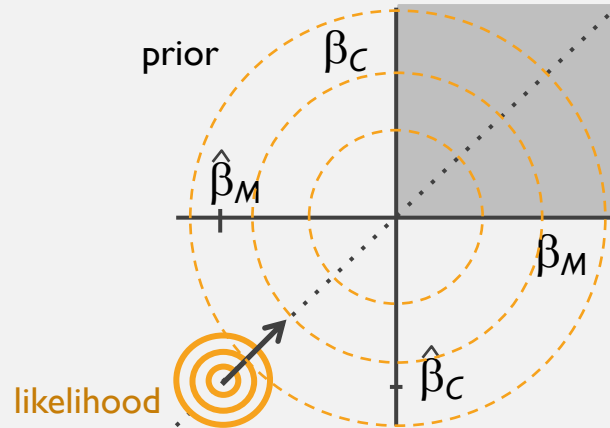
- Output for hypothetical Data A.

	Data A			
	c_t	f_t	$\text{BF}(M_p M_u)$	PMP
H_1	1/4	1	4	0.27
H_2	1/2	1	2	0.13
H_3	1/8	1	8	0.53
H_{unc}	1	1	1	0.07



COMPARISON BAYES FACTOR VS POSTERIOR PROBABILITY

- Consider $H_1: \beta_M > 0, \beta_C > 0$ versus H_2 : “not H_1 ”.
- Compare $\Pr(H_1|\text{data})$ with $\Pr(\beta > \mathbf{0}|\text{data})$.



USEFUL PROPERTIES OF BAYES FACTORS

- **Bayes factors...**
 - ... have an intuitive interpretation as the **relative evidence in the data** between statistical hypotheses.
 - ... can be used for testing **multiple equality and order hypotheses**.
 - ... do **not rely on large sample theory**.
 - ... do **not rely on the sampling plan**.
 - ... are **consistent** in very general scenario's

SOFTWARE FOR BAYES FACTOR TESTING

- **BIEMS** (Mulder et al., 2012) for testing hypotheses with equality and order constraints in the (multivariate) t test, (multivariate) regression, (M)AN(C)OVA, repeated measures .
- BIEMS has a user-friendly interface.
- R-package in development.



SOFTWARE FOR BAYES FACTOR TESTING

BIEMS

☒ Step 1 ☒ Step 2 ☐ Step 3 ☐ Step 4

Data Input Models Generate Default Prior Calculate Bayes factors

Select or Generate Model Specification Generate Default Prior Edit Default Prior Calculate Bayes factors

BIEMS - Bayesian inequality and equality constrained model selection.
By Joris Mulder.

Specify Models

1: Compose

Variables

$\mu(1,1)$
 $\alpha(1,1)$
 $\alpha(2,1)$
 $\alpha(3,1)$

7 8 9 +
4 5 6 >
1 2 3 <
0 . =

Backspace

Add to list

2: List of Inequalities and Equalities

$\alpha(1,1) > \alpha(2,1)$
 $\alpha(2,1) > \alpha(3,1)$
 $\alpha(3,1) > 0$

Edit Delete Define as Model

3: List of Models

Model 2

Edit Model Delete Model

4: Settings

Prior Restrictions CECPP

☒ Equivalent to model constraints
☐ Specify restrictions manually Model Restriction Mode

Scale of prior variance

☒ Number of free parameters under the restrictions
☐ Manually:

MCMC sample size: 20000
Max BF steps for models with equalities: 30

Standardize

Variable	Yes	No
DV1	<input type="checkbox"/>	<input checked="" type="checkbox"/>
EV1	<input type="checkbox"/>	<input checked="" type="checkbox"/>
EV2	<input type="checkbox"/>	<input checked="" type="checkbox"/>
EV3	<input type="checkbox"/>	<input checked="" type="checkbox"/>

Step Back Close

BIEMS!



SOFTWARE FOR BAYES FACTOR TESTING

- **BOCOR** (Mulder, 2016) and **BCT** (Mulder & Gelissen, in prep.) for testing equality and order constraints on bivariate correlations, partial correlations, polychoric correlations, etc.
- BOCOR and BCT are stand-alone executables.
- R-package in development.



SOFTWARE FOR BAYES FACTOR TESTING

```
Input 1: model
#DV    #covs    intercept    #populations    Ntotal
3      3        1          1          1286

Which DVs are ordinal (0=continuous, 1=ordinal)
1 1 1

Input 2: hypotheses
#hypotheses
2
#equalities, #inequalities
1 1
0 2

Input 3: constraints in hypotheses
Equalities H1; Inequalities H1; Equalities H2; Inequalities H2; etc.
1 2 1 1 3 1
1 3 1 1 3 2

1 2 1 0 -1 0
1 3 2 0 1 0

Input 4: implementation details
iterations, iseed, delta
1000 123 .1
```

BOCOR/BCT!

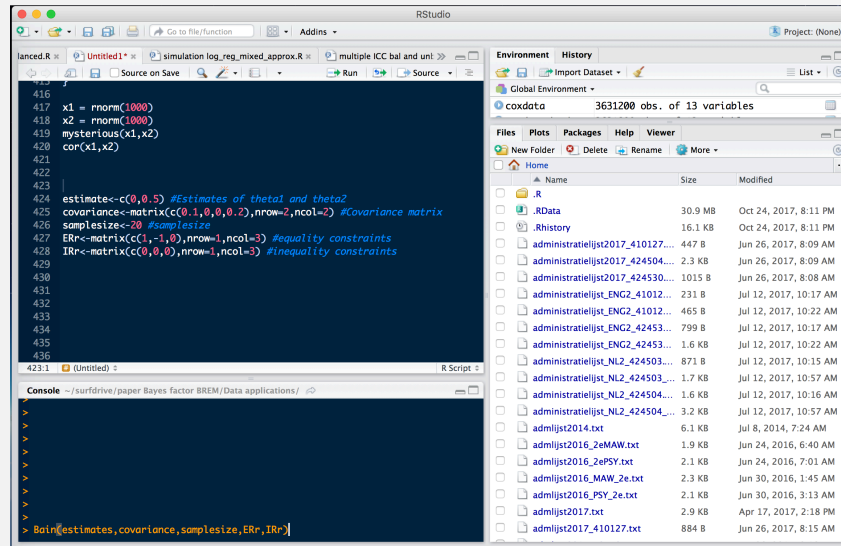


SOFTWARE FOR BAYES FACTOR TESTING

- **Bain** (Gu et al., in press) for testing equality and order constraints on parameters for **general statistical models**.
 - Bain uses an approximate Bayes procedure that only needs the ML estimates and estimated covariance matrix.
- Bain is a R-package. It is currently being implemented in JASP (Wagenmakers and colleagues)



SOFTWARE FOR BAYES FACTOR TESTING



The screenshot shows the RStudio environment with the following code in the editor:

```
416  
417 x1 = rnorm(1000)  
418 x2 = rnorm(1000)  
419 mysterious(x1,x2)  
420 cor(x1,x2)  
421  
422  
423  
424 estimate=c(0,0.5) #Estimates of theta1 and theta2  
425 covariance=matrix(c(0.1,0,0,0.2),nrow=2,ncol=2) #Covariance matrix  
426 sampleSize=20 #sampleSize  
427 ERr<-matrix(c(1,-1,0),nrow=1,ncol=3) #equality constraints  
428 IRr<-matrix(c(0,0,0),nrow=1,ncol=3) #inequality constraints  
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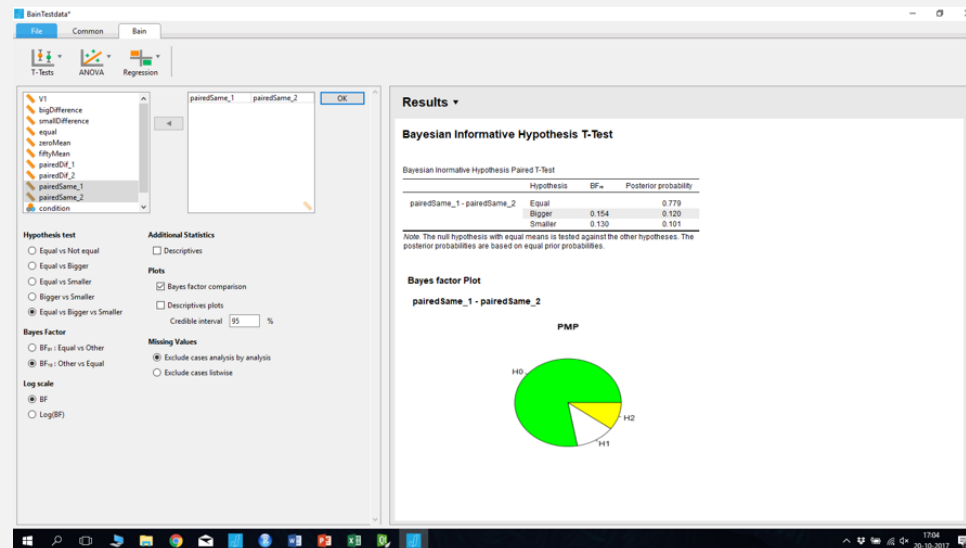
The console shows the following command:

```
> Bain(estimate,covariance,sampleSize,ERr,IRr)
```

Bain in R!



SOFTWARE FOR BAYES FACTOR TESTING



Bain in JASP!



SOFTWARE FOR BAYES FACTOR TESTING

- Other software packages include:
 - **BFvar** (Böing-Messing et al., 2017) for testing equality and order constraints on **group variances**.
 - **BF-ICC** (Mulder & Fox, under review) for testing constraints on **intraclass correlations**.
 - **BF-NAM** (Dittrich et al., in prep.) for testing constraints on **network autocorrelations**.
- Bayes factor testing not for order constraints:
 - **BayesFactor** R-packge (Morey & Rouder, 2015)
 - **JASP** (Wagenmakers and colleagues, 2016).
 - **SPSS** (latest version).



ORDER HYPOTHESES ON RELATIVE EFFECTS REVISITED

- We can combine these order constraints into a single 'order hypothesis'.
- "All sources of workplace aggression have a positive effect on depression and the effect is largest for managers, followed by coworkers, followed by visitors"
- Data: N = 864.

Power differential hypothesis: $H: \beta_1 > \beta_2 > \beta_3 > 0$

Analysis in **BIEMS**

$BF(H_1, H_u) = 9.1$

$BF(H_2, H_u) = .46$



$BF(H_1, H_2) = 19.8$

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS REVISITED

- **Null hypothesis:**

$$H_0: \rho_{NL} = \rho_{CR} = \rho_{GER} = \rho_{DEN}.$$

- **Order hypothesis:**

$$H_1: \rho_{NL} < \rho_{CR} < \rho_{GER} < \rho_{DEN}.$$

- **Complement hypothesis:**

$$H_2: \text{neither } H_0, \text{ nor } H_1.$$

- Data: 100-150 schools per county; school classes of 15 students.

- **Results:**

Analysis in R (Mulder & Fox, under review)

$$BF(H_0, H_u) = .000$$

$$BF(H_1, H_u) = 18.1$$

$$BF(H_2, H_u) = .261$$



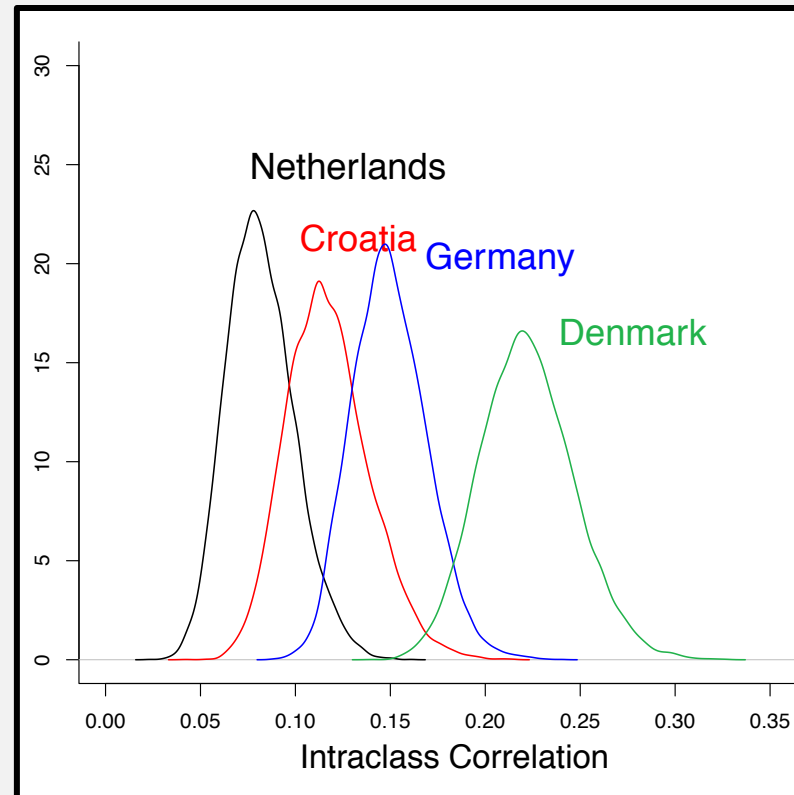
$$P(H_0|Data) = .000$$

$$P(H_1|Data) = .986$$

$$P(H_2|Data) = .014$$

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS REVISITED

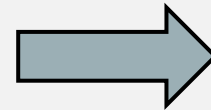
- **Posteriors** under unconstrained model



ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS REVISITED

- It is often suggested that information-sharing occurs sooner and at a higher rate among colleagues who they feel related to – this is often attributed to identity.

- $H_1: \beta_{\text{position}} > \beta_{\text{building}} > \beta_{\text{gender}} > 0$
- $H_2: \beta_{\text{position}} > \beta_{\text{building}} = \beta_{\text{gender}} = 0$
- $H_3: \text{neither } H_1, \text{ nor } H_2.$

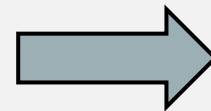


$$\begin{aligned} P(H_1|\text{Data}) &= .06 \\ P(H_2|\text{Data}) &= .94 \\ P(H_3|\text{Data}) &= .00. \end{aligned}$$

ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS REVISITED

- It is often suggested that information-sharing occurs sooner and at a higher rate among colleagues who they feel related to – this is often attributed to identity.
- Data: 1505 emails among 51 colleagues.

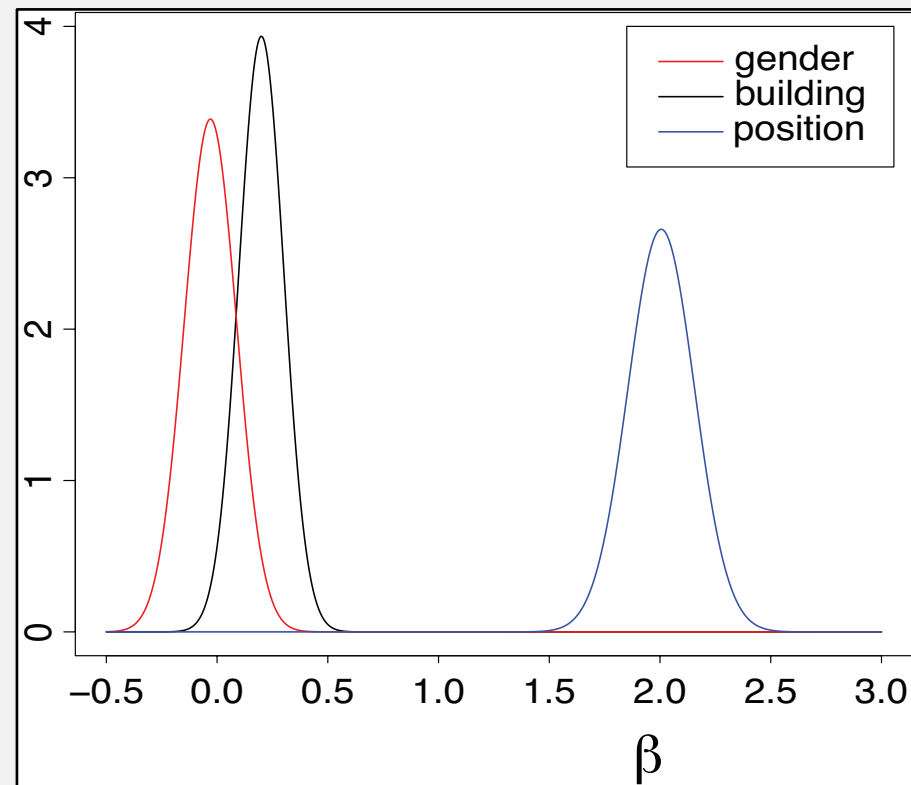
- $H_1: \beta_{\text{position}} > \beta_{\text{building}} > \beta_{\text{gender}} > 0$
- $H_2: \beta_{\text{position}} > \beta_{\text{building}} = \beta_{\text{gender}} = 0$
- H_3 : neither H_1 , nor H_2 .



$$\begin{aligned} P(H_1|\text{Data}) &= .06 \\ P(H_2|\text{Data}) &= .94 \\ P(H_3|\text{Data}) &= .00. \end{aligned}$$

ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS REVISITED

- **Posteriors** under unconstrained model



CONCLUSIONS

- The Bayes factor has many useful properties for testing statistical hypotheses such as
 - its **intuitive interpretation** as the statistical evidence between competing hypotheses;
 - the ability to test **multiple hypotheses** with **equality and order constraints**;
 - its **consistent** behavior or their invariance for the **sampling plan**.
- **Software** for computing Bayes factors is becoming more and more available **for challenging testing problems**.

THANK YOU!

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