BAYESIAN TESTING OF EQUALITY AND ORDER-CONSTRAINED HYPOTHESES:

APPLICATIONS, METHODOLOGY & SOFTWARE

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TILBURG UNIVERSITY

OVERVIEW

- Applications of equality and order constrained hypothesis testing
  - Linear regression in organizational psychology
  - Multilevel modeling in educational measurement
  - Relational event modeling of dynamic social networks
- Classical approaches
- Bayes factors
  - Basics, properties, behavior
- Software for Bayes factor testing of constrained hypotheses
- Applications revisited

JOINT WORK WITH…

- Herbert Hoijtink (Utrecht University);
- Roger Leenders (Tilburg University);
- Jean-Paul Fox (University of Twente);
- Eric-Jan Wagenmakers (University of Amsterdam);
- Irene Klugkist (Utrecht University);
- Gu Xin (University of Liverpool);
- Dino Dittrich (Tilburg University);
- Florian Boïng-Messing (Tilburg University);
- Johan Braeken (University of Oslo);
- Stephen Wood (University of Leicester).










TESTING COMPOSITE HYPOTHESES WITH EQUALITY AND ORDER CONSTRAINTS

- The magnitude of statistical parameters are **not absolute** but **relative**.
- Effects are not only **relative to each other**, but also to the **area of the science**, and to the research method that is employed (Cohen, 1988).

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- For example, a (standardized) effect of .3 may be large when regressing **political left-right placement** on **educational level**, but small when **regressing quality-of-life** on **treatment dosage**.
- Therefore test composite hypotheses with **equality** (=) and **order constraints** (<, >) on the effects of interest.

ORDER HYPOTHESES ON RELATIVE EFFECTS

- **Workplace aggression** from inside and outside the organization.
- Employees at hospitals indicated whether they experienced any form of aggression by **managers**, **coworkers**, and/or **visitors**.
- The degree of **anxiety**, **depression**, and **job dissatisfaction** was measured for each respondent which served as dependent variables.

ORDER HYPOTHESES ON RELATIVE EFFECTS

- **Relative effects of independent variables on a dependent variable** are generally modeled using regression models.
- An example of workplace aggression on workers' well-being.

$$\text{Depression} = \beta_0 + \beta_V \times \text{Aggression(V)} + \beta_C \times \text{Aggression(C)} + \beta_M \times \text{Aggression(M)} + \text{error}$$

- with β_V = Relative effect of aggression from **visitors** on depression.

  β_C = Relative effect of aggression from **colleagues** on depression.

  β_M = Relative effect of aggression from **managers** on depression.

ORDER HYPOTHESES ON RELATIVE EFFECTS

- Hypotheses are often formulated with order constraints.
  - We expected a **positive effect** of aggression from visitors on depression, i.e., $\beta_V > 0$.
  - We expected a **positive effect** of aggression from coworkers on depression, i.e., $\beta_C > 0$.
  - We expected a **positive effect** of aggression from managers on depression, i.e., $\beta_M > 0$.
  - Based on the **hierarchy in the organization** we expected that the relative effect of aggression on depression is largest for managers, followed by coworkers, followed by visitors: $\beta_M > \beta_C > \beta_V$.
- See Braeken et al. (2014).

ORDER HYPOTHESES ON RELATIVE EFFECTS

- We can combine these order constraints into a single 'order hypothesis'.
  - "All sources of workplace aggression have a positive effect on depression and the effect is largest for managers, followed by coworkers, followed by visitors":

Power differential hypothesis: H_1: $\beta_M > \beta_C > \beta_V > 0$

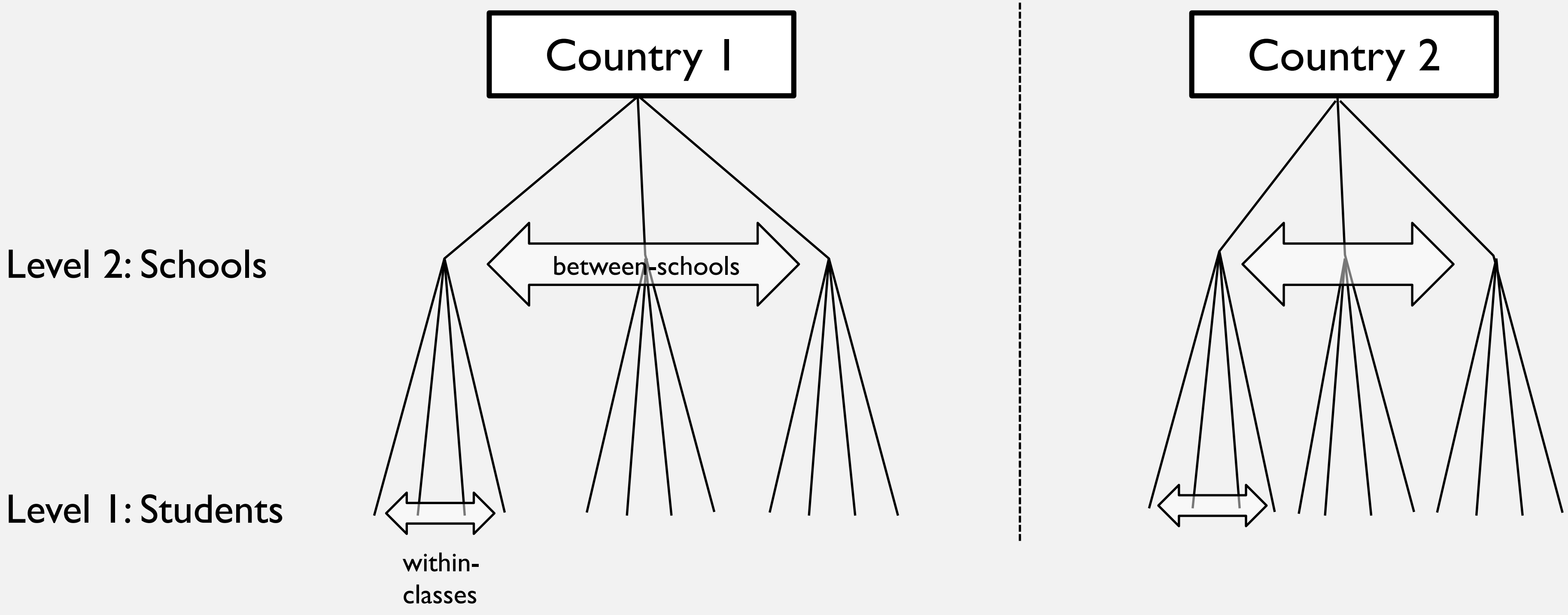
versus

Complement hypothesis: H_2: "not H_1".

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS

- The Trends in International Mathematics and Science Study (**TIMSS**) measures the performances of fourth and eight graders in more than 50 participating countries around the world (www.iea.nl/timss).
- Interest is not only in testing the average grades across countries, but also in testing the **variability of school performance within countries**.
- The relative variability between schools is quantified by **country specific intraclass correlations**.

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS



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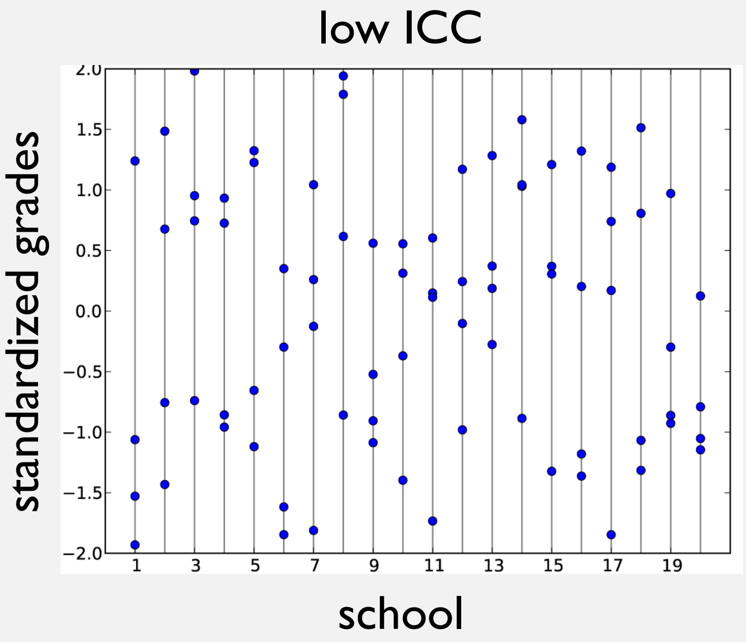
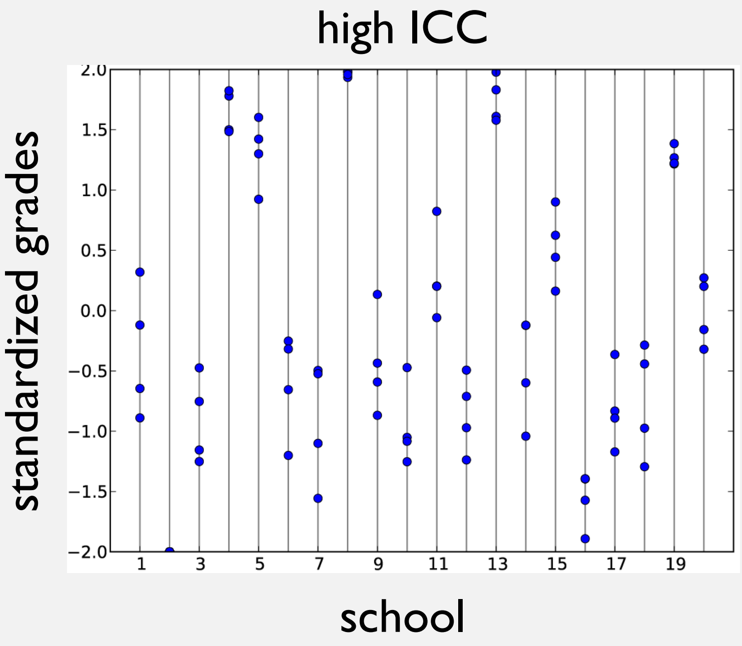




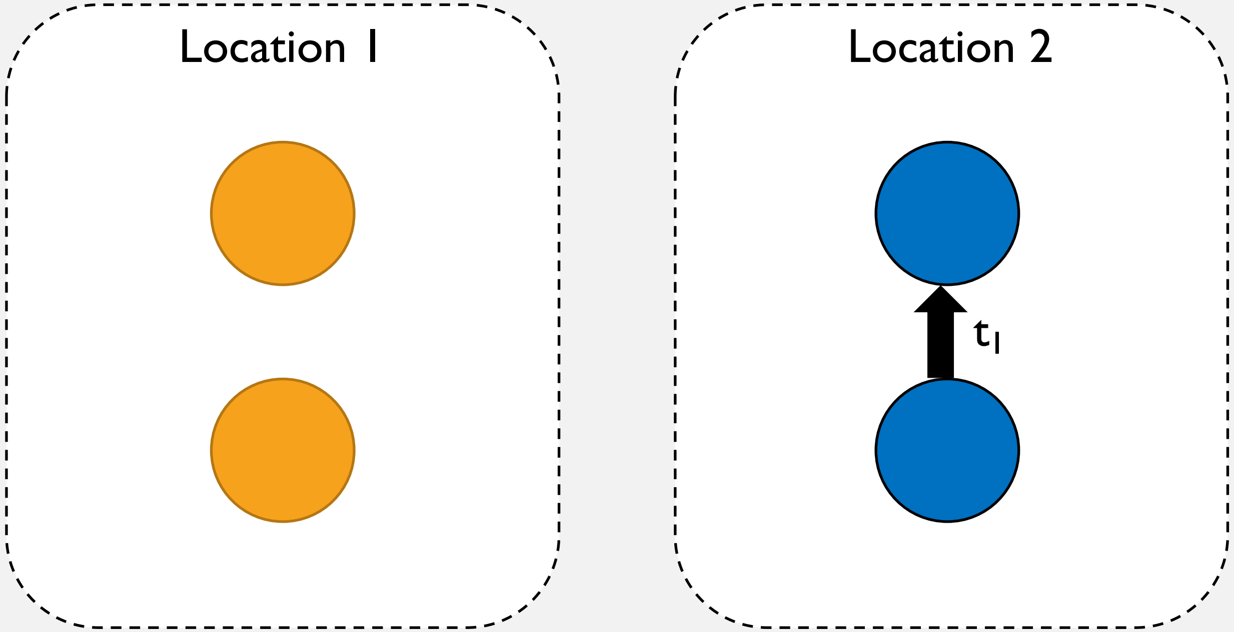
ORDER HYPOTHESES ON INTRACLASS CORRELATIONS

- Focus on The Netherlands, Croatia, Germany, and Denmark.
- **Null hypothesis**: Equal intraclass correlations over countries:
  - H_0: $\rho_{NL} = \rho_{CR} = \rho_{GER} = \rho_{DEN}$.
- **Order hypothesis**: Previous research suggest that between schools variability is largest for Danish schools, followed by German schools, followed by Croatian schools, followed by Dutch Schools:
  - H_1: $\rho_{NL} < \rho_{CR} < \rho_{GER} < \rho_{DEN}$.
- **Complement hypothesis**:
  - H_2: neither H_0, nor H_1.

ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS

- A **relational event history data** was collected of email messages between colleagues containing information about innovative ideas in a large consultancy firm.
- The data contain information about **who** send a message **to whom** and **when** in the year 2010.
- Interest was in the **relative importance** of drivers of these social interactions between colleagues and how this changes over time.
- The relational event model of Butts (2008) was employed.

ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS



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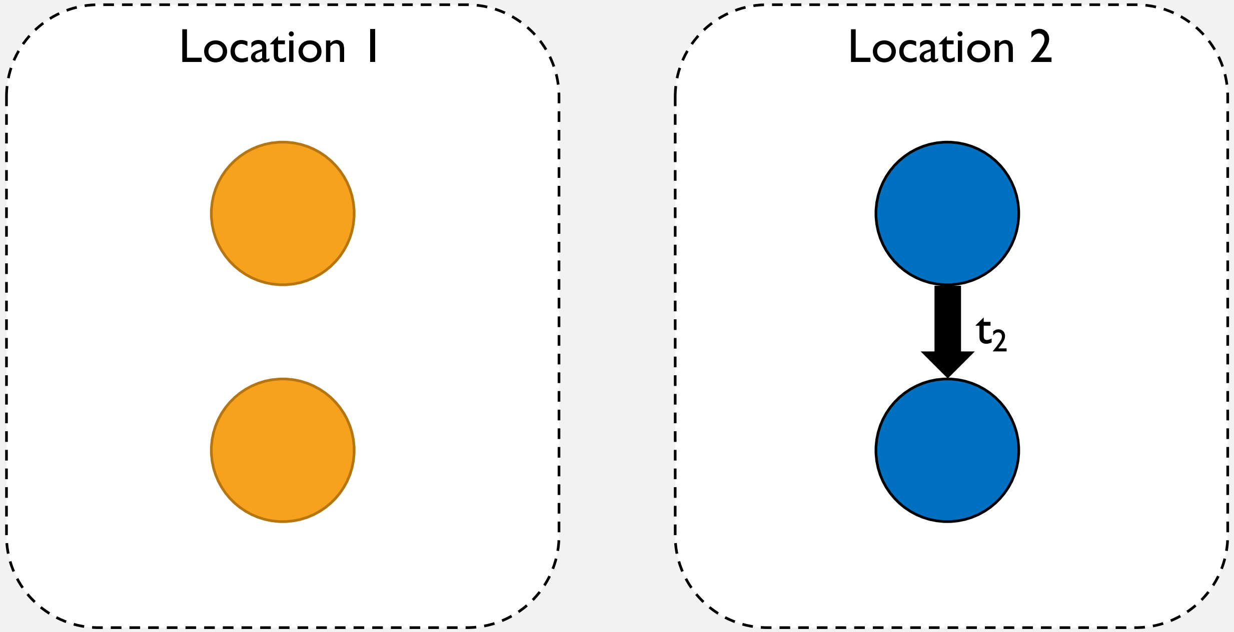

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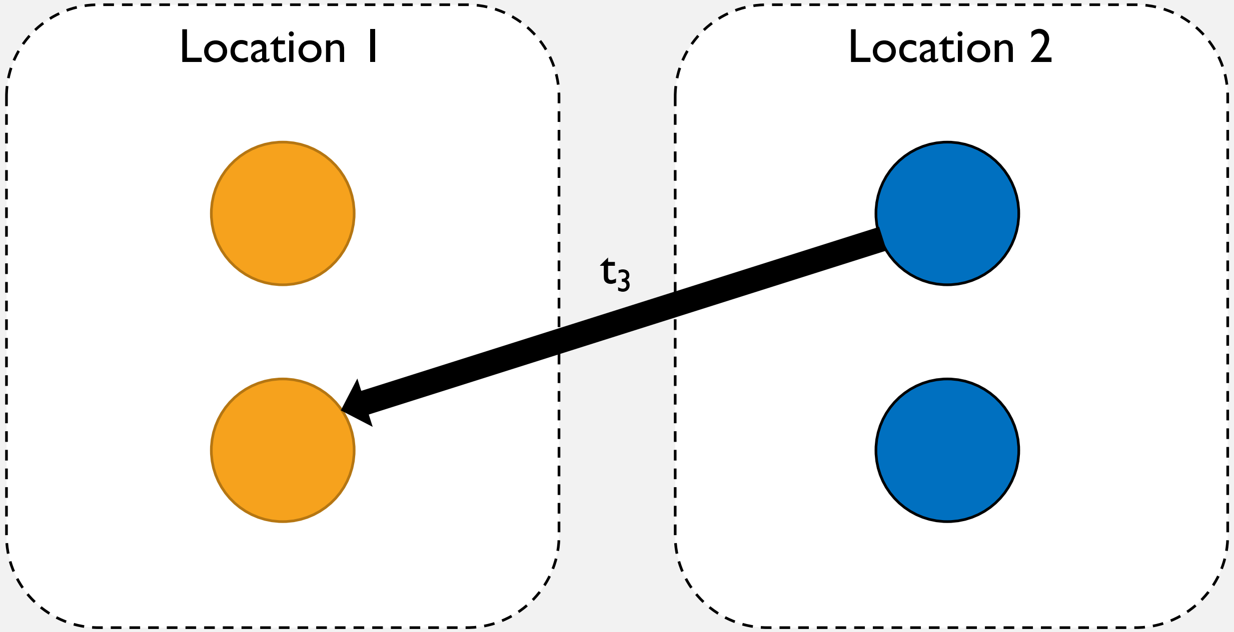

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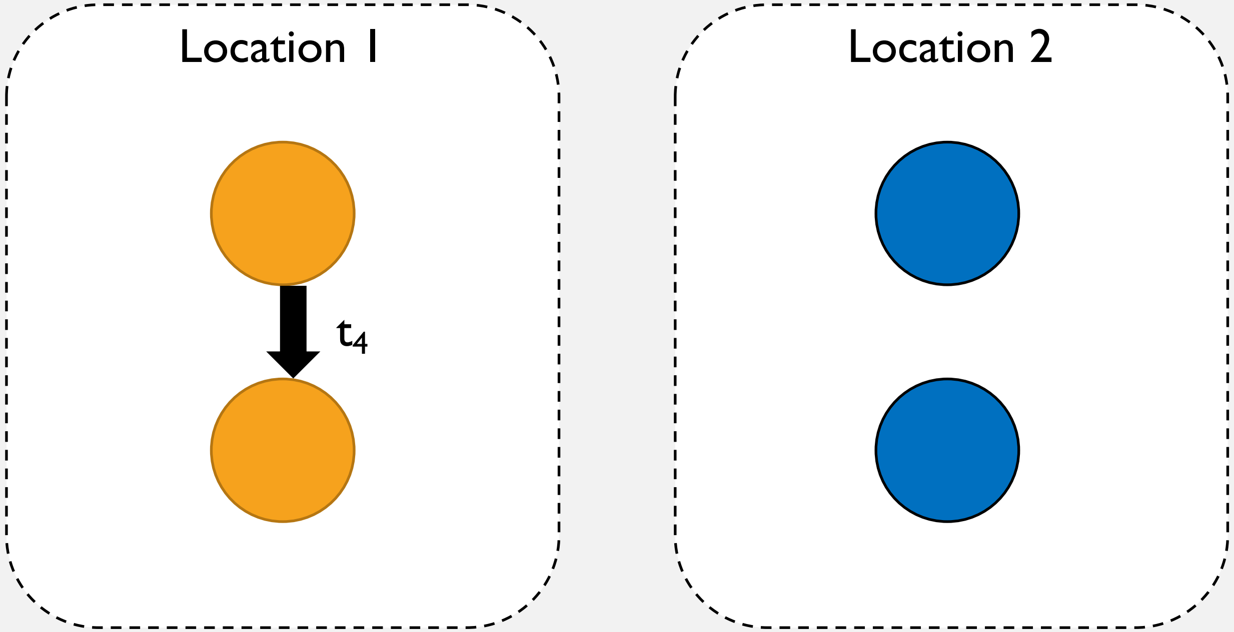

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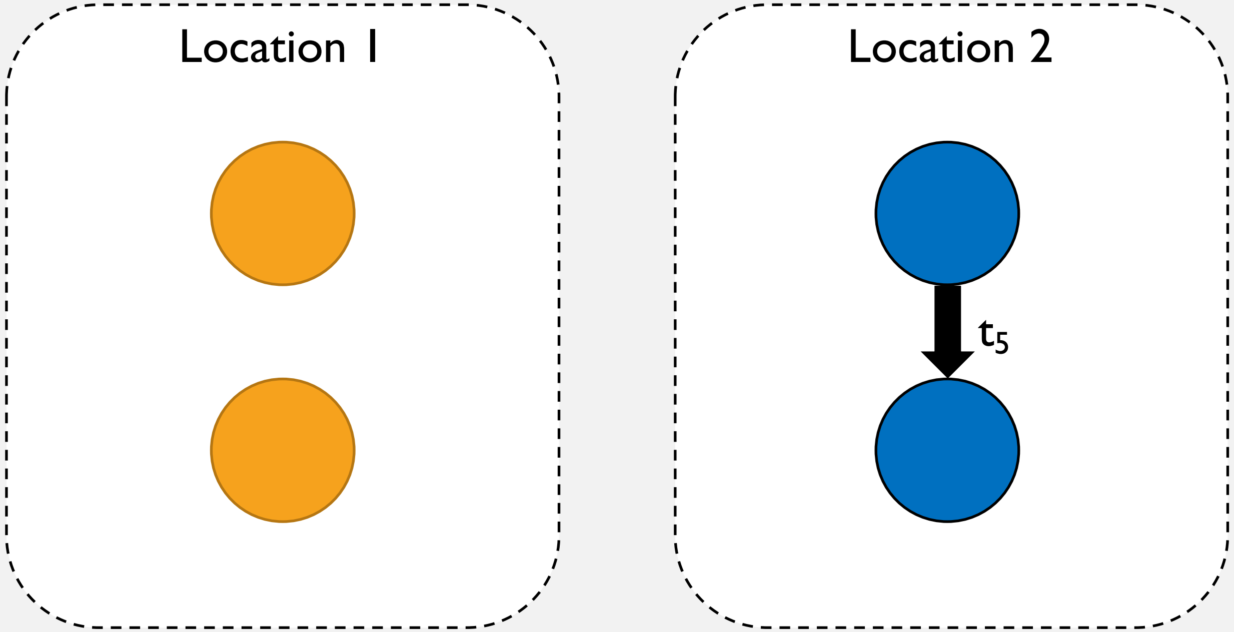

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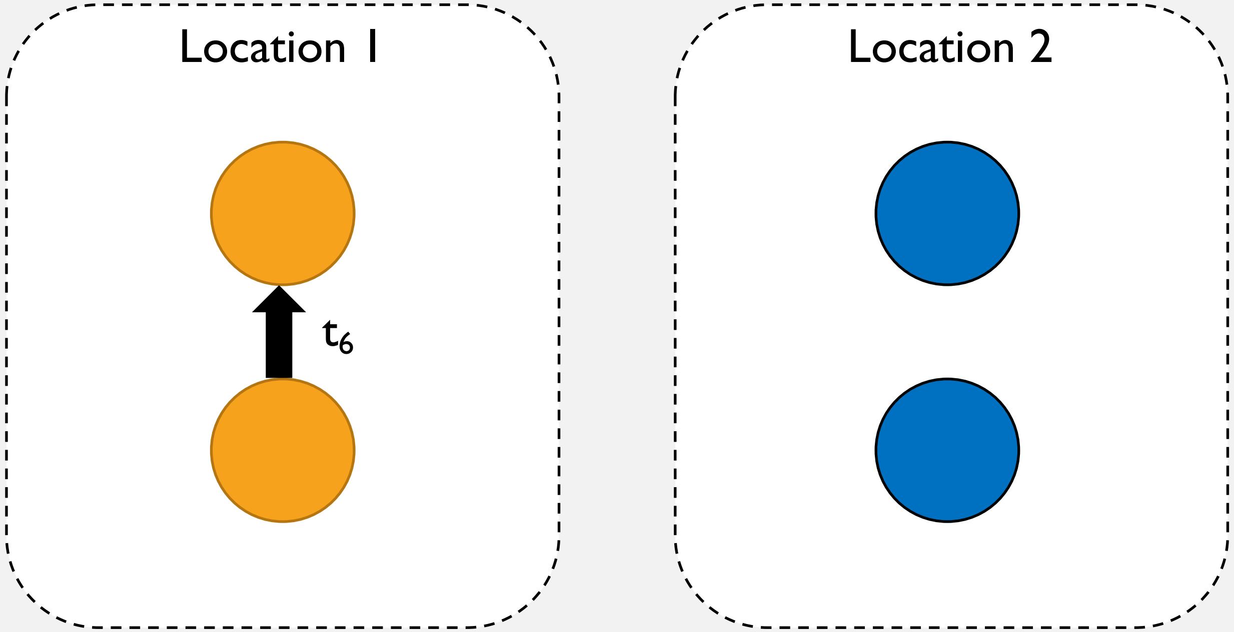

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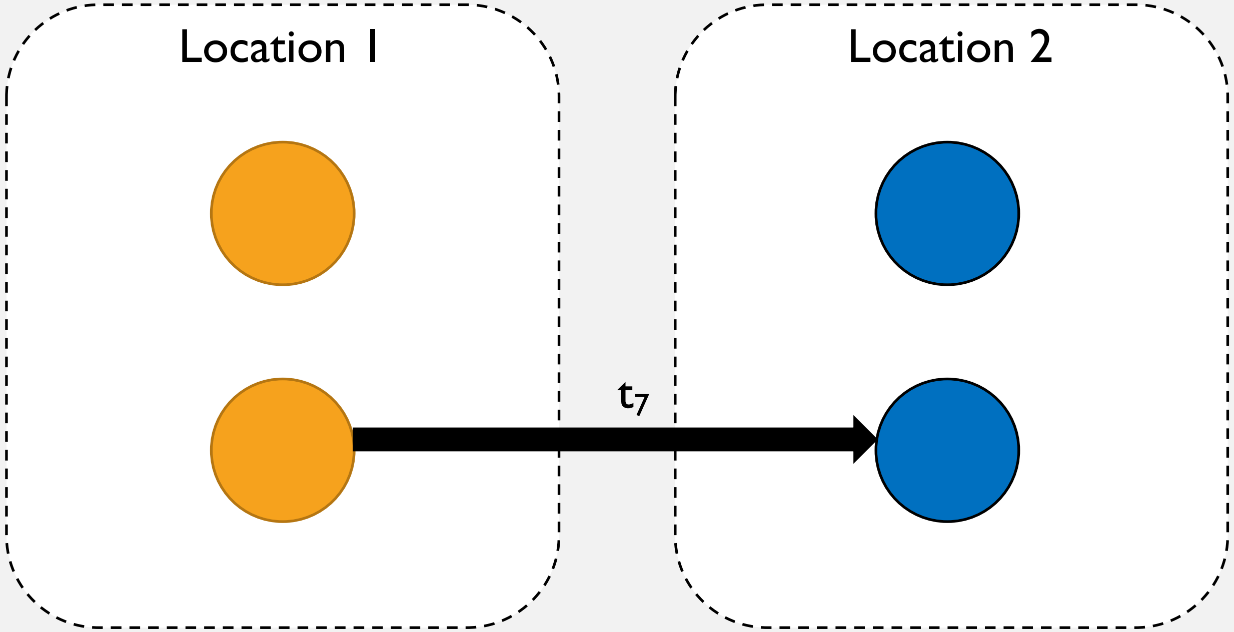

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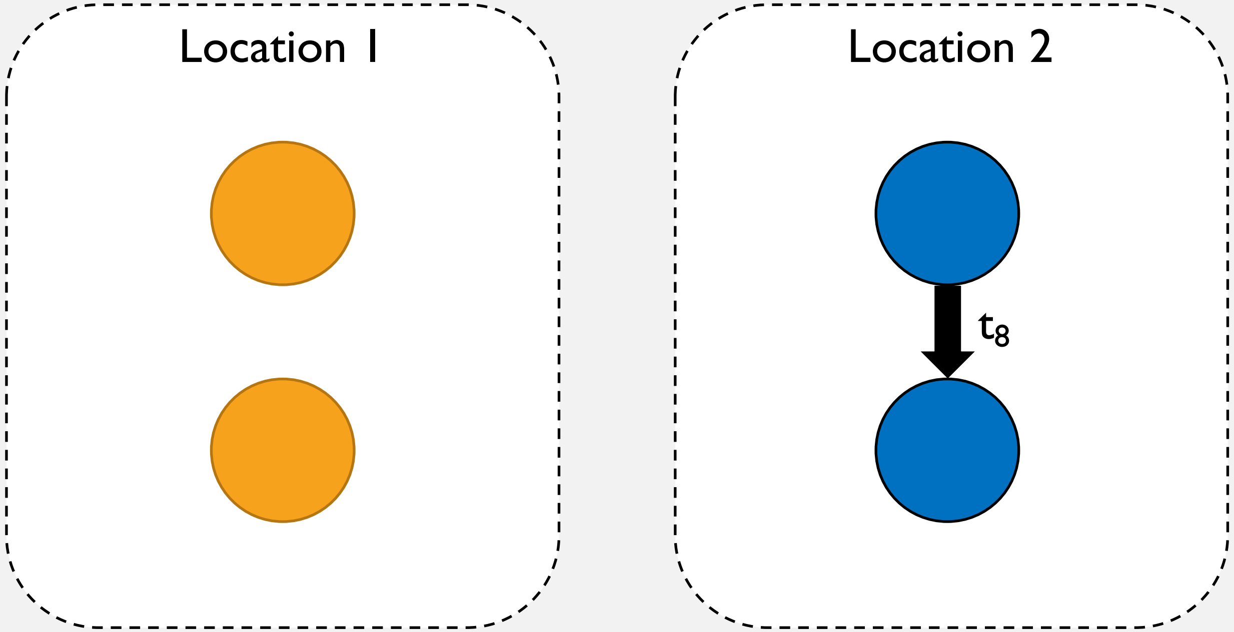

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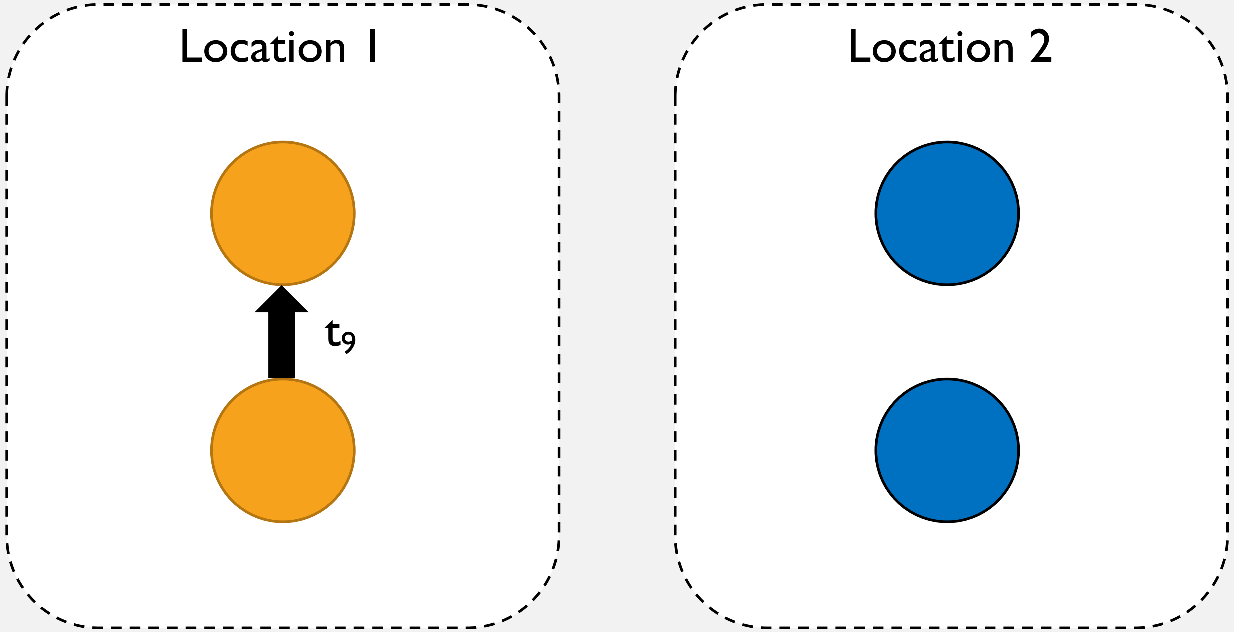

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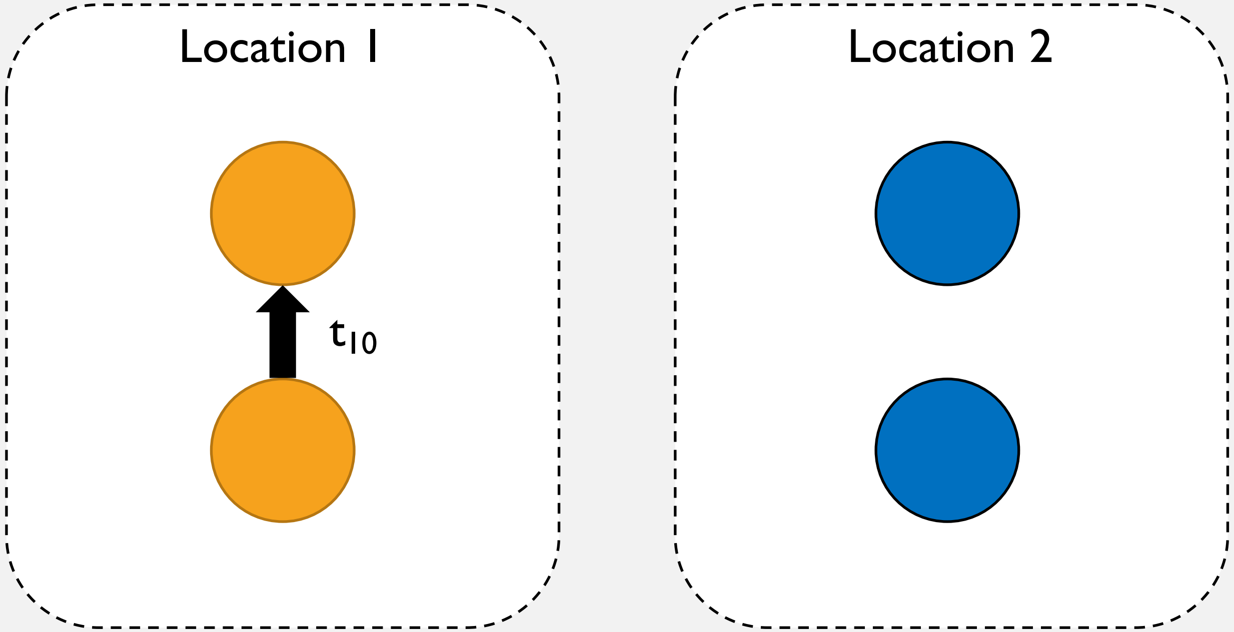

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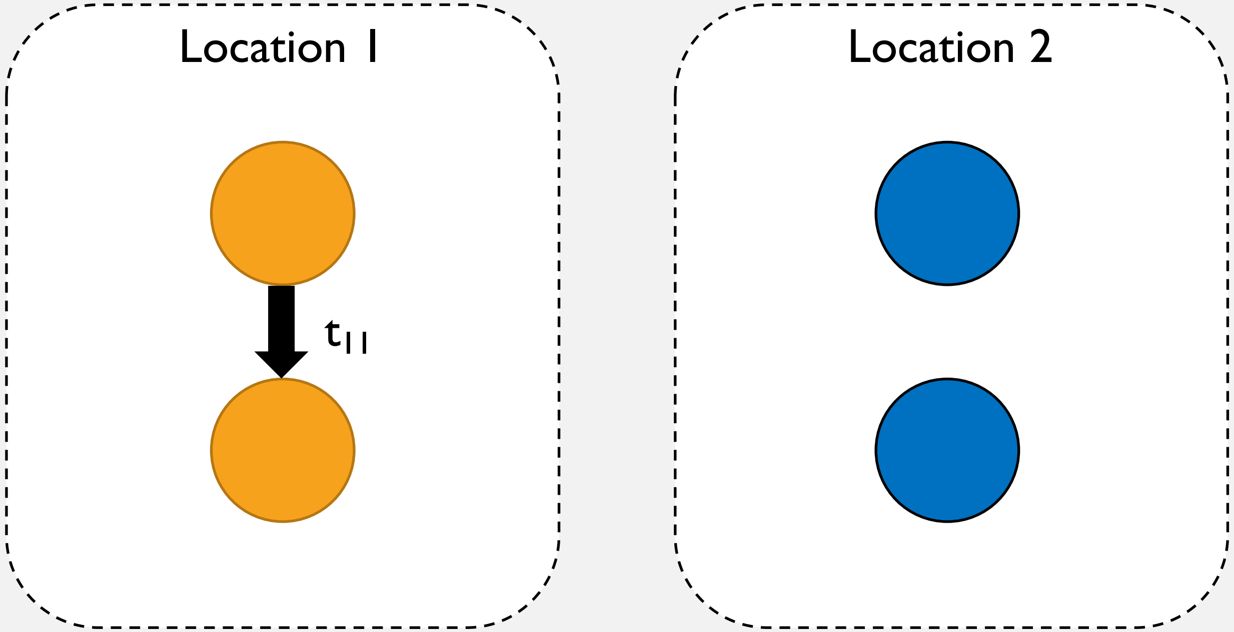

ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS

- It is often suggested that information-sharing occurs sooner and at a higher rate among colleagues who they feel related to – this is often attributed to **identity**.
- Thus, having the same position may have a positive and strong effect on information sharing, while working in the same building, or being of the same gender may also have a positive effect but to a lesser extent.
  - H_1: $\beta_{position} > \beta_{building} > \beta_{gender} > 0$
- Alternatively, only being in the same location may have an effect.
  - H_2: $\beta_{position} > \beta_{building} = \beta_{gender} = 0$
  - H_3: neither H_1, nor H_2.

WHY TEST ORDER HYPOTHESES?

- Why test order hypotheses instead of the classical null and alternative hypotheses?
  - We test the magnitude of the **effects relative to the scientific field and study**.
  - We get a **direct answer** whether our theory or expectations (with order constraints) are supported by the data.
  - We test with more statistical **power**.
    - Compare the two-tailed test (“$\beta \neq 0$”) with the one-tailed test (“$\beta > 0$”).
    - More power implies that we are more likely to draw the correct conclusions.

HOW CAN WE TEST ORDER HYPOTHESES?

- **Significance tests using p-value**…
  - …are only available for testing **specific types of order hypotheses** for 'standard' models (Silvapulle & Sen, 2004);

H_0: $\beta_1 = \beta_2 = \beta_3$ against H_1: $\beta_1 > \beta_2 > \beta_3$	YES
H_0: $\beta_1 > \beta_2 > \beta_3$ against H_1: $\beta_1 , \beta_2 , \beta_3$	YES
H_0: $\beta_1 = \beta_2 > \beta_3$ against H_1: $\beta_1 , \beta_2 , \beta_3$	NO
H_0: $\beta_1 > \beta_2 > \beta_3$ against H_1: $\beta_1 < \beta_2 < \beta_3$	NO

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$\beta_1 \neq 0$ (sign.), $\beta_2 = 0$ (not sign.), $\beta_1 = \beta_2$ (not sign.).

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  - …are **inconsistent** when the null is true;

$$H_0: \beta_1 = \beta_2 = \beta_3 \text{ against } H_1: \beta_1 > \beta_2 > \beta_3$$

For extremely large samples there is still a probability of α to incorrectly reject a true H_0.

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  - …are not designed for simultaneously testing **multiple (more than 2) hypotheses**.

H_0: $\beta_1 > \beta_2 > \beta_3$ vs H_1: $\beta_1 < \beta_2 < \beta_3$ vs H_2: neither H_0, nor H_1.

HOW CAN WE TEST ORDER HYPOTHESES?

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  - …depend on the **sampling plan**.

HOW CAN WE TEST ORDER HYPOTHESES?

- **(Classical) information criteria**, such as the AIC or BIC, are not designed for testing models with order hypotheses on the parameters (Mulder et al., 2009).
  - AIC = $2d - 2\log(\hat{L})$

Model complexity
(number of free parameters)

Model fit
(maximized likelihood)

- How many **free parameters** *d* does H_1: $\beta_M > \beta_C > \beta_V > 0$ have?

BAYES FACTORS

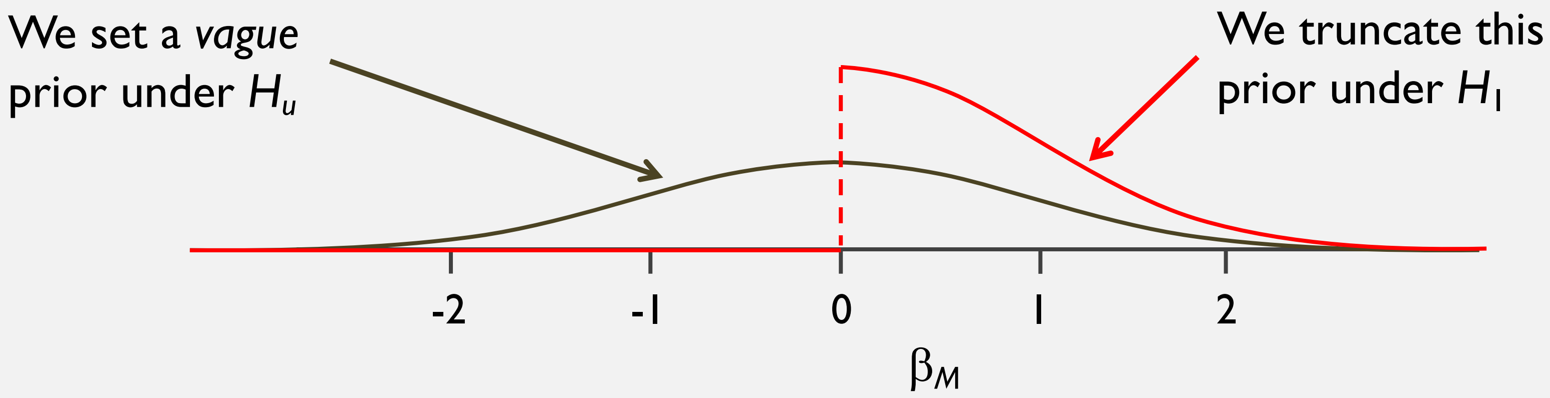
- The **Bayes factor** (Jeffreys, 1961) is defined as the ratio of the marginal likelihoods

$$BF(H_1,H_2) = \frac{\text{marginal likelihood}(H_1)}{\text{marginal likelihood}(H_2)}$$

- The **marginal likelihood** of H_1 quantifies how plausible the data were generated under hypothesis H_1.
- For example, a Bayes factor of BF(H_1,H_2) = 10 implies that it was 10 times more plausible that the data were generated under H_1 than under H_2.
- Marginal likelihoods can be **difficult to compute**.
- Bayes factors can be **sensitive to the prior**.

BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

- When testing order hypotheses we can use the *encompassing prior* approach to set priors under the hypotheses (e.g., Klugkist et al., 2005).
- Consider H_1: $\beta_M > 0$ versus H_u: β_M unconstrained,



BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

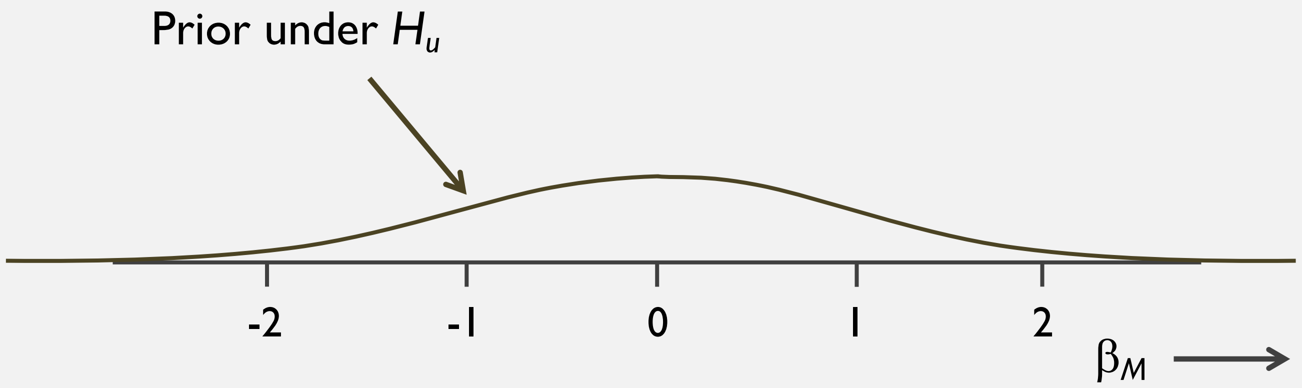
- The Bayes factor of an order hypothesis H_1 against the unconstrained hypothesis H_u is then equal to

$$BF(H_1,H_u) = \frac{\text{posterior probability that the constraints of } \mathrm{H}_1 \text{ hold under } H_u}{\text{prior probability that the constraints of } \mathrm{H}_1 \text{ hold under } H_u}$$

$$= \frac{\text{Relative fit of } \mathrm{H}_1}{\text{Relative complexity of } \mathrm{H}_1}$$

BAYES FACTORS FOR TESTING A ONE-SIDED HYPOTHESIS

- For example, H_1: $\beta_M > 0$ versus H_u: β_M unconstrained,



- $\text{BF}(H_1,H_u) = \dfrac{\text{posterior probability that } \beta_M > 0}{\text{prior probability that } \beta_M > 0}$

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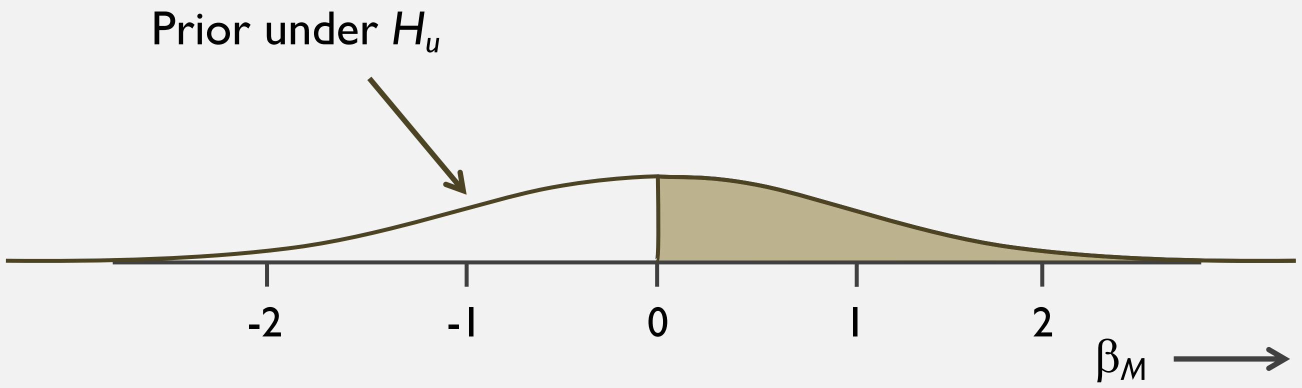

- $\text{BF}(H_1,H_u) = \dfrac{\text{posterior probability that } \beta_M > 0}{\text{prior probability that } \beta_M > 0} = \dfrac{\quad}{0.5}$

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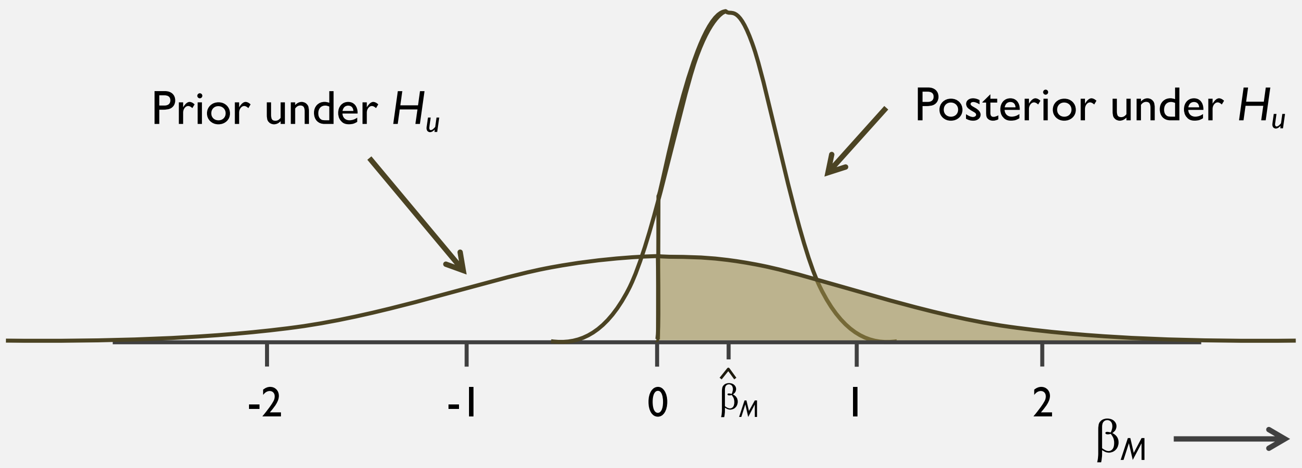

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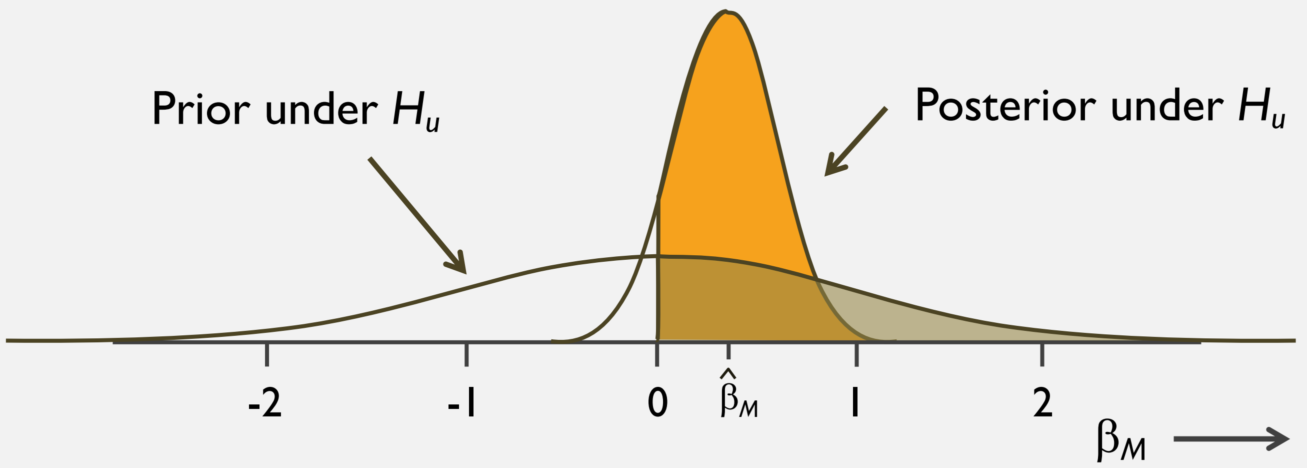
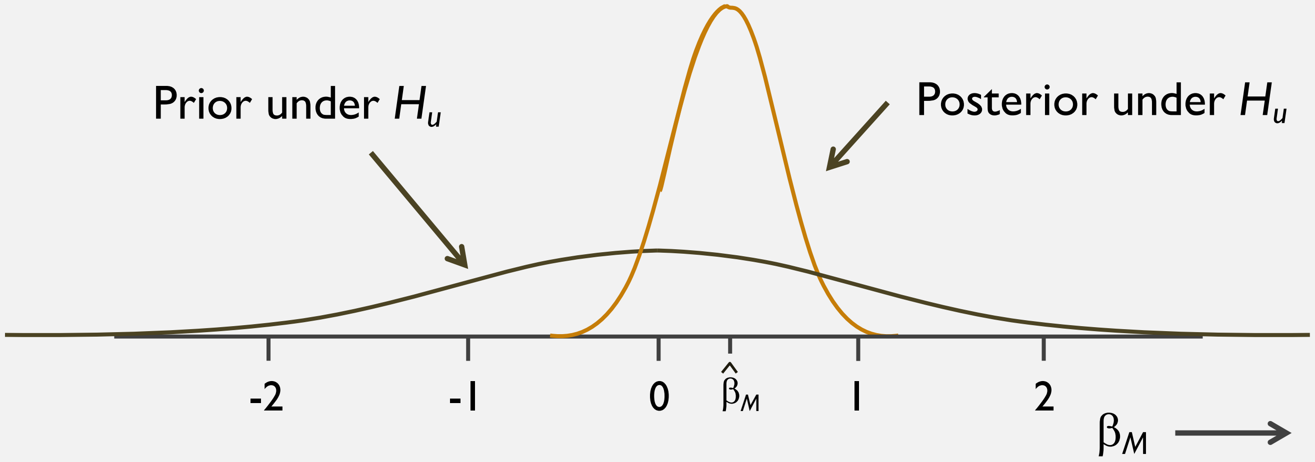


- $$\mathrm{BF}(H_1,H_u) = \frac{\text{posterior probability that } \beta_M > 0}{\text{prior probability that } \beta_M > 0} = \frac{0.9}{0.5} = 1.8$$

BAYES FACTORS FOR TESTING A PRECISE HYPOTHESIS

- For example, H_0: $\beta_M = 0$ versus H_1: $\beta_M \neq 0$,



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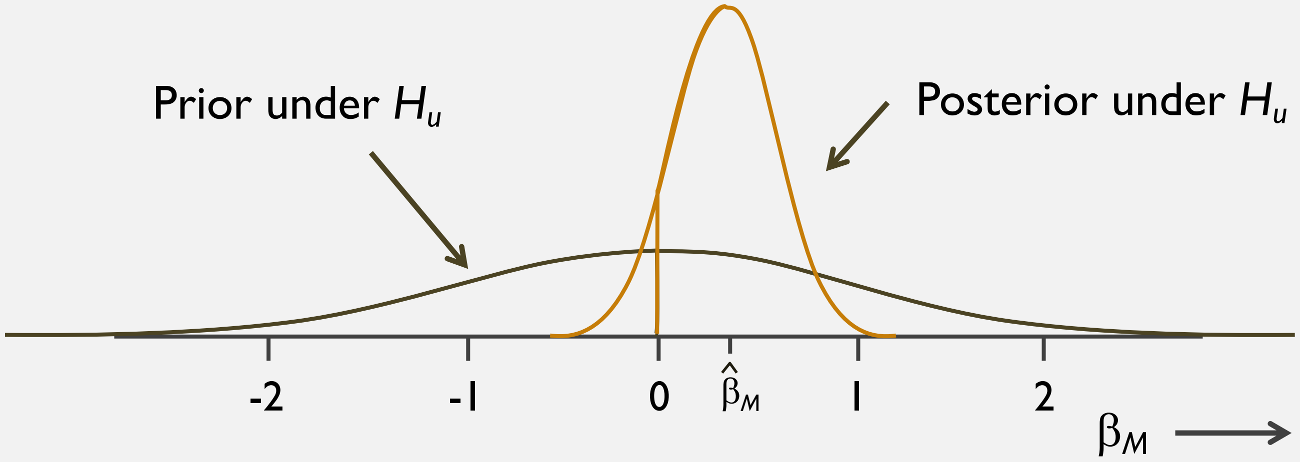

- BF(H_0,H_1) = $\dfrac{\text{posterior density at } \beta_M\text{=0}}{\text{prior density at } \beta_M\text{=0}}$

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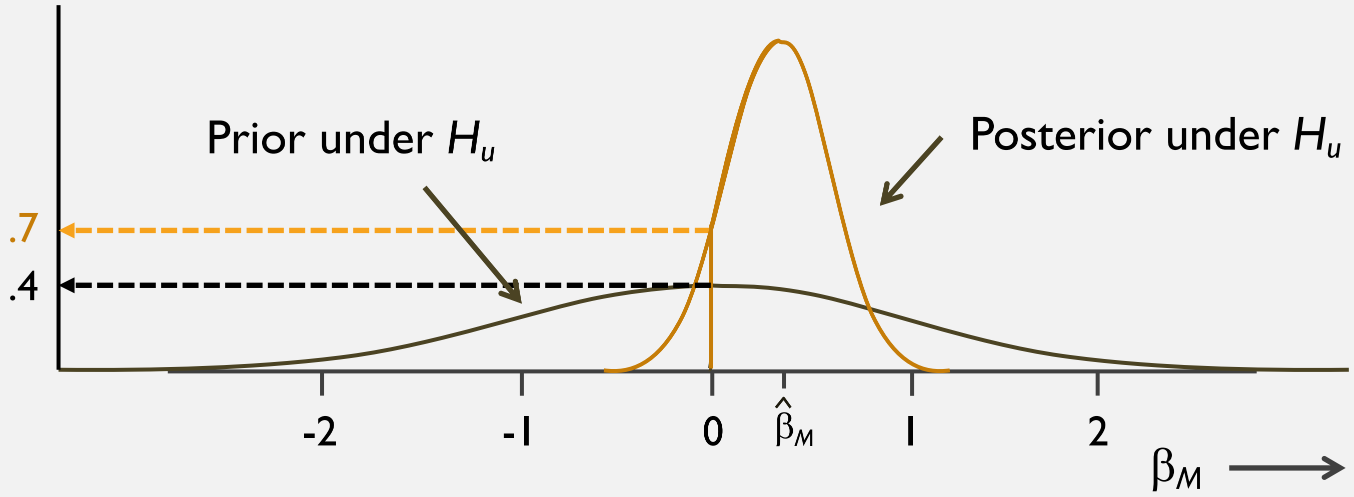

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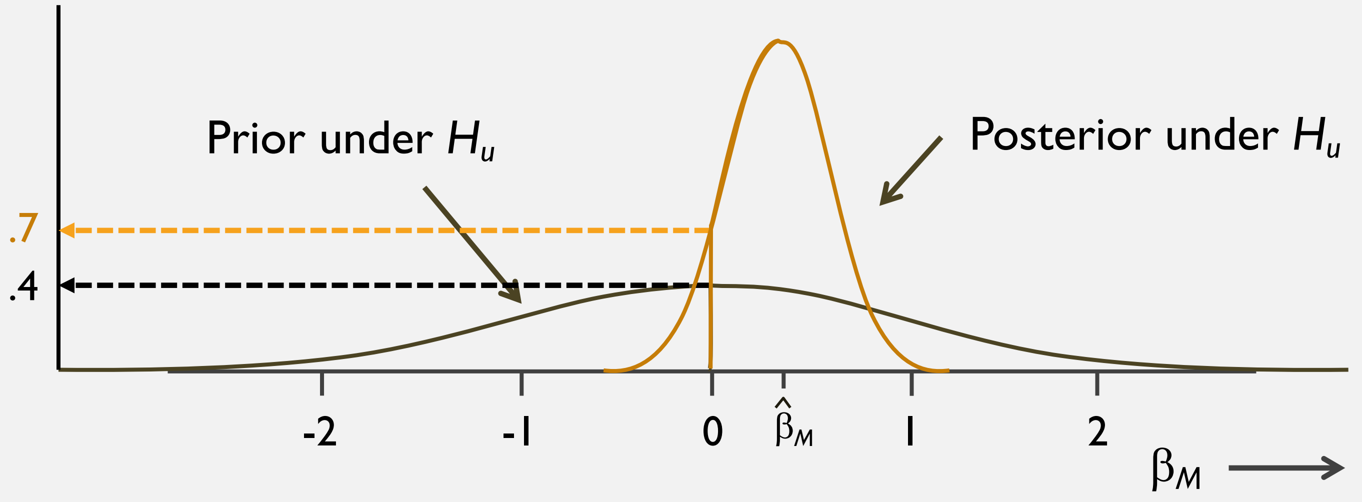

- $$BF(H_0,H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0} = \frac{.7}{.4} = 1.75$$

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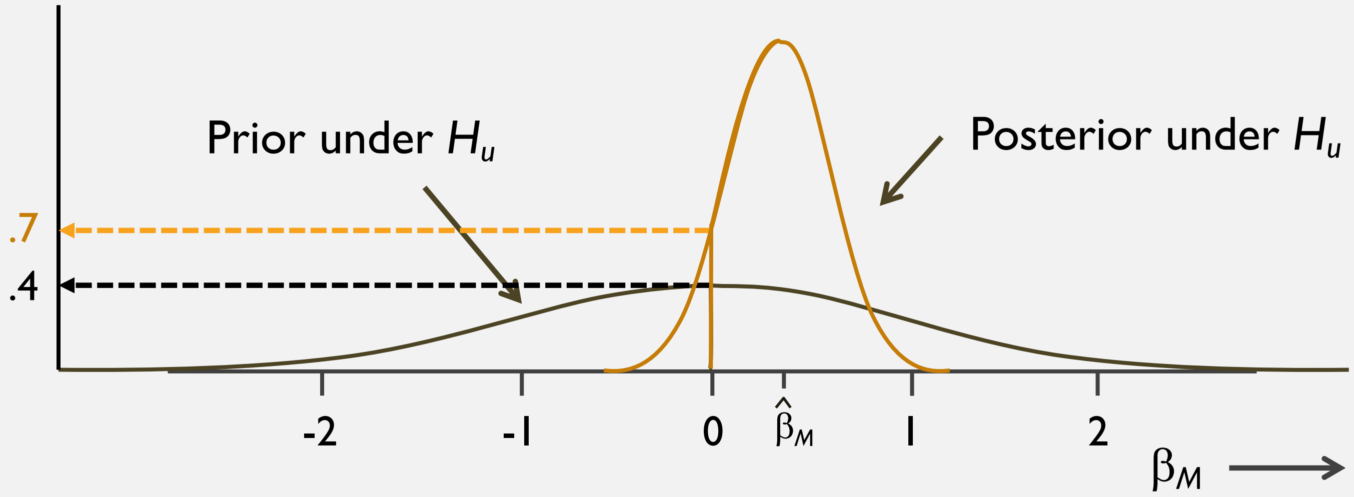

- $\text{BF}(H_0,H_1) = \dfrac{\text{posterior density at } \beta_M\text{=0}}{\text{prior density at } \beta_M\text{=0}} = \dfrac{.7}{.4} = 1.75$

The Savage-Dickey density ratio of the Bayes factor holds for a specific choice of the priors.

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- **Bartlett's paradox**: The evidence for a null hypothesis H_0: $\beta_M = 0$ against H_1: $\beta_M \neq 0$ can be made arbitrarily large by setting the prior variance large enough.

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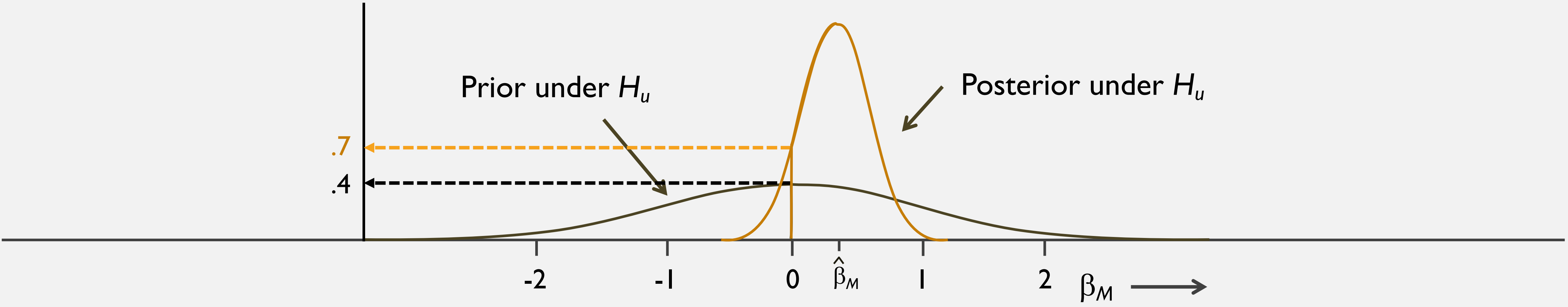

$$BF(H_0, H_1) = \frac{\text{posterior density at } \beta_M = 0}{\text{prior density at } \beta_M = 0} = \frac{.7}{.4} = 1.75$$

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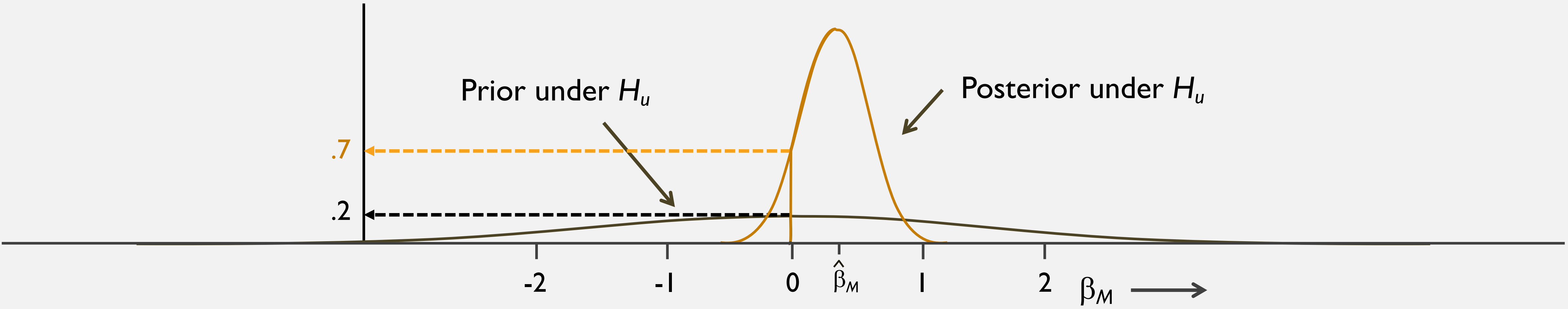

$$BF(H_0,H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0} = \frac{.7}{.2} = 3.50$$

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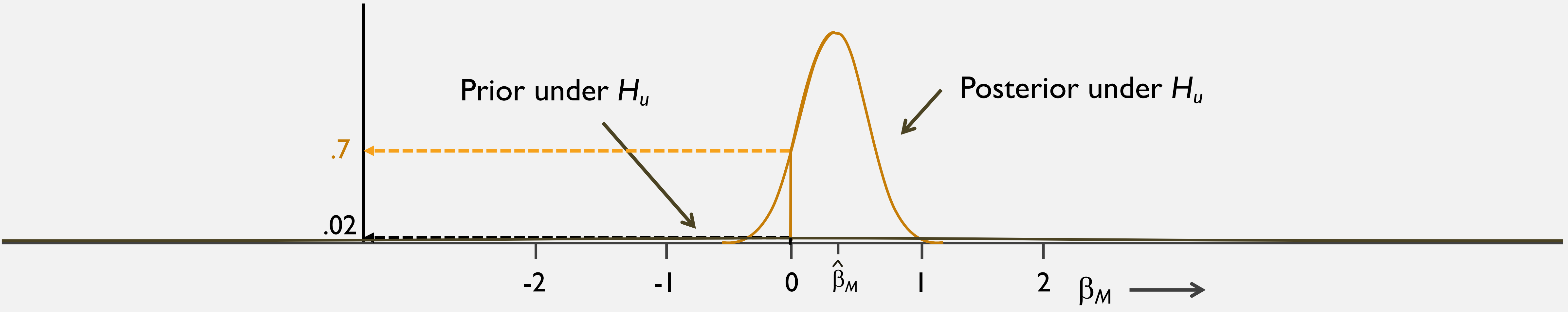

$$BF(H_0,H_1) = \frac{\text{posterior density at } \beta_M=0}{\text{prior density at } \beta_M=0} = \frac{.7}{.02} = 35$$

SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

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- If clear prior information is absent use a default prior that contains the **information of a minimal experiment**.

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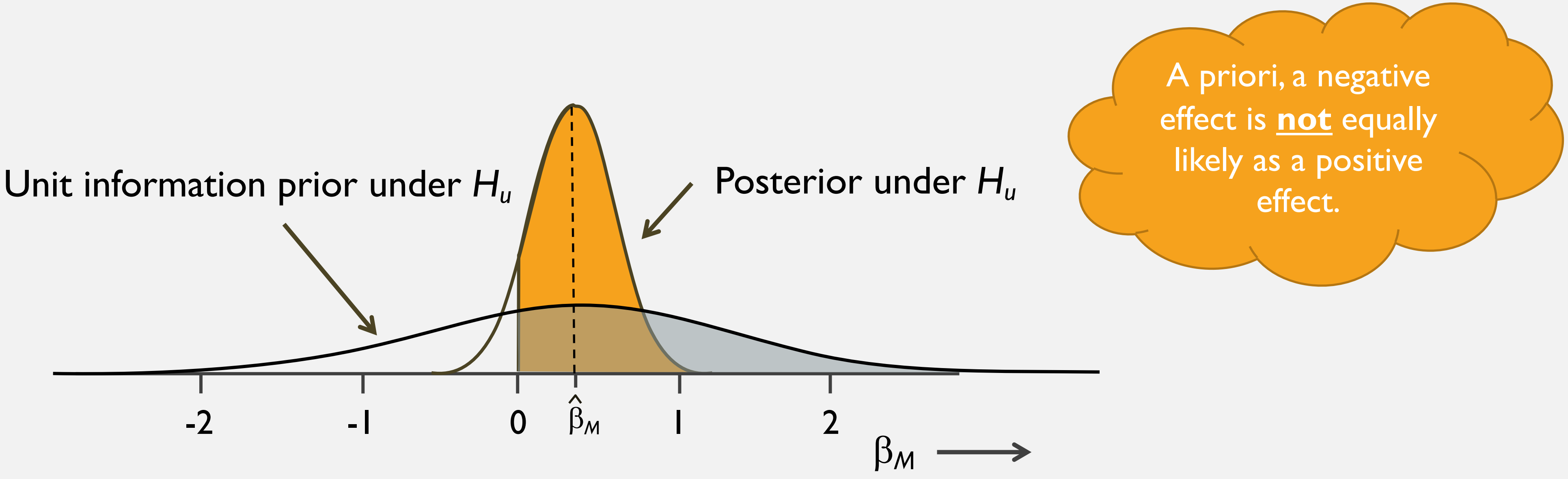
- If clear prior information is absent use a default prior that contains the **information of a minimal experiment**.
- The **BIC** is a Bayes factor approximation based on a **unit information prior** (Schwarz, 1978; Raftery, 1995).

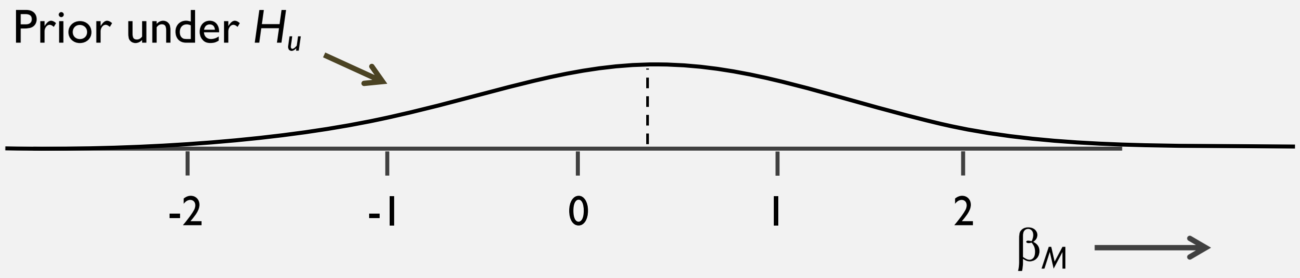
SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- The **unit information prior** as centered at the ML estimate.



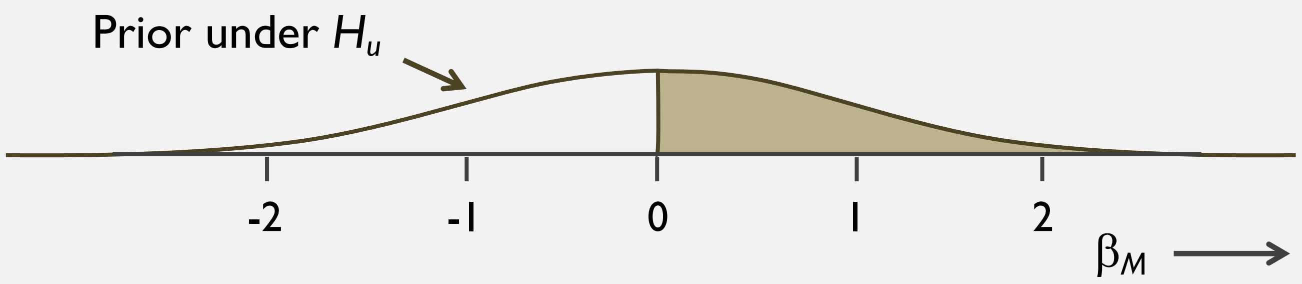
SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- A default prior should be **centered at the null value**. Then



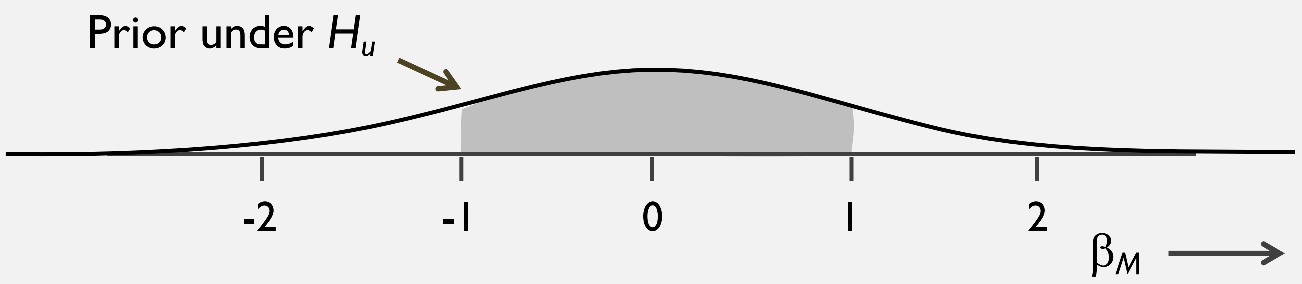
SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- A default prior should be **centered at the null value**. Then
  - **negative effects** are equally likely as **positive effects**;



SENSITIVITY OF THE BAYES FACTOR TO THE PRIOR

- A default prior should be **centered at the null value**. Then
  - **negative effects** are equally likely as **positive effects**;
  - **small effects** are more likely a priori than **large effects**.



POSTERIOR ODDS OF HYPOTHESES

- The Bayes factors can be used to update the **prior odds** of two hypotheses to obtain the **posterior odds** between the two hypotheses.

$$\frac{\mathrm{P}(H_1 \mid \text{data})}{\mathrm{P}(H_2 \mid \text{data})} = \mathrm{BF}(H_1, H_2) \times \frac{\mathrm{P}(H_1)}{\mathrm{P}(H_2)}$$

posterior odds — Bayes factor — prior odds

BAYES FACTORS FOR TESTING ORDER HYPOTHESES

- Example of multiple hypothesis test
  - H_1: $\beta_M > 0, \beta_C > 0$ (both effects are positive)
  - H_2: $\beta_M > \beta_C$ (effect of managers is larger than effect of coworkers)
  - H_3: $\beta_M > \beta_C > 0$ (combination of H_1 and H_2)
  - H_u: β_M, β_C (no specific expectation on the effects)

BAYES FACTORS FOR TESTING ORDER HYPOTHESES

- Example of multiple hypothesis test
  - H_1: $\beta_M > 0, \beta_C > 0$
  - H_2: $\beta_M > \beta_C$
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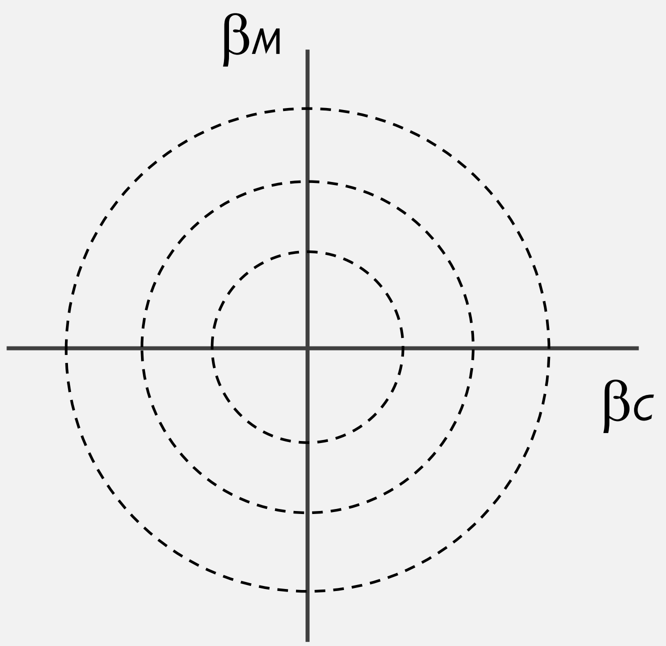
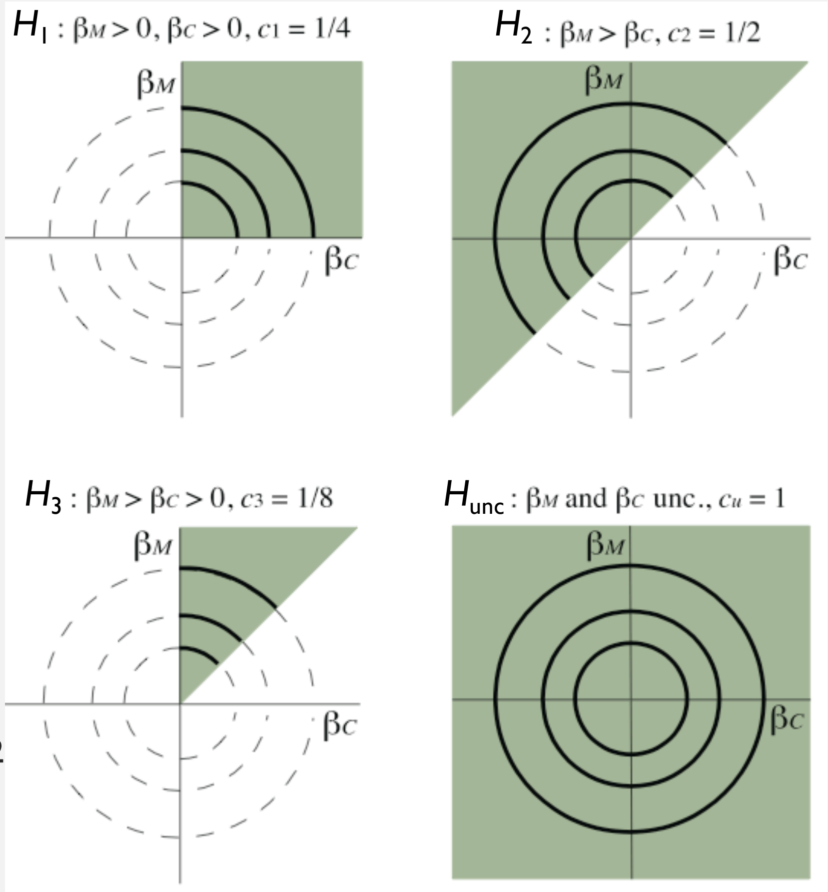
  - H_u: β_M, β_C



A vague prior for β_C and β_M under H_u.

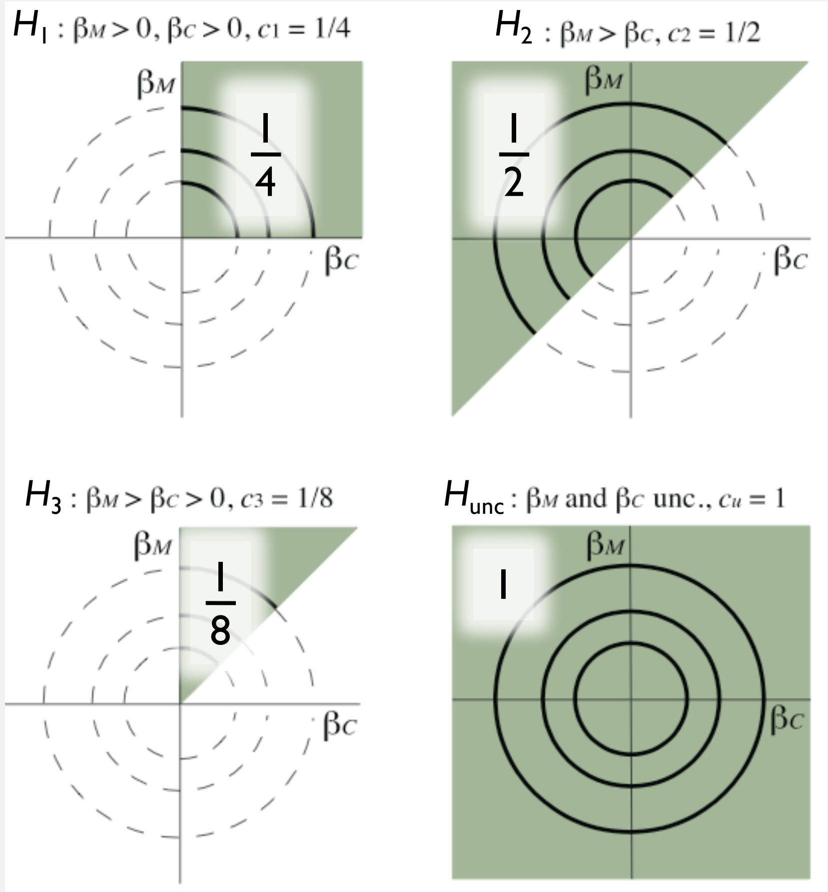
RELATIVE COMPLEXITY OF ORDER HYPOTHESES

- Example of multiple hypothesis test
  - H_1: $\beta_M > 0, \beta_C > 0$
  - H_2: $\beta_M > \beta_C$
  - H_3: $\beta_M > \beta_C > 0$
  - H_u: β_M and β_C
- So H_u is **most complex (largest parameter space)**, followed by H_2, H_1, and H_3 (smallest parameter space).



RELATIVE COMPLEXITY OF ORDER HYPOTHESES

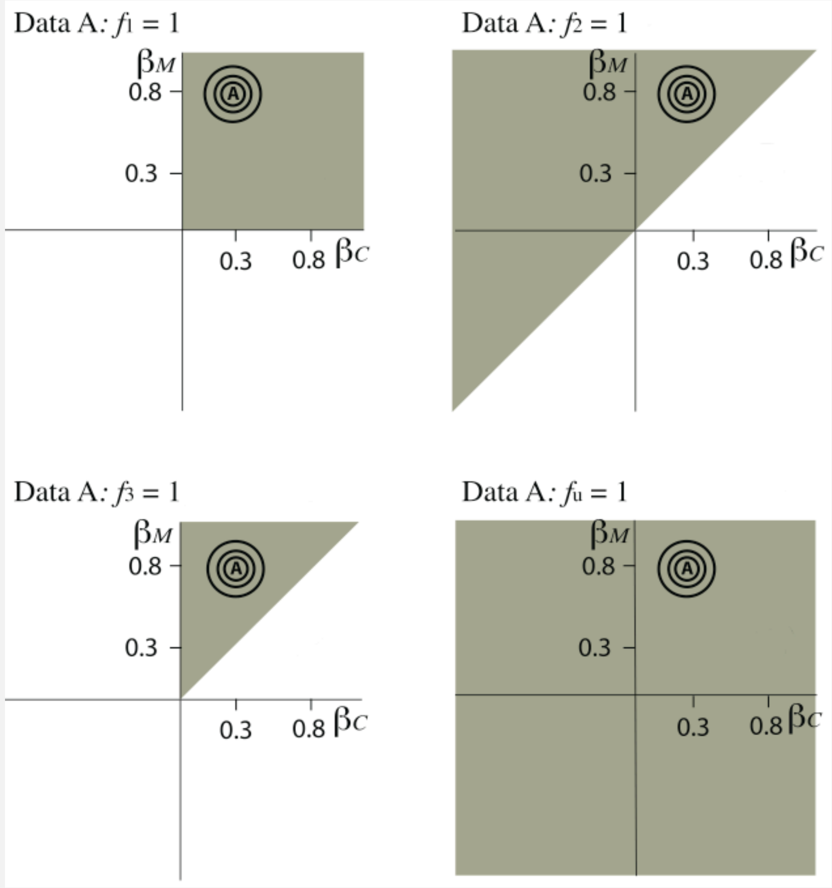
- Example of multiple hypothesis test
  - H_1: $\beta_M > 0, \beta_C > 0$
  - H_2: $\beta_M > \beta_C$
  - H_3: $\beta_M > \beta_C > 0$
  - H_u: β_M and β_C
- **Relative complexity** (Mulder et al., 2010).



RELATIVE COMPLEXITY OF ORDER HYPOTHESES

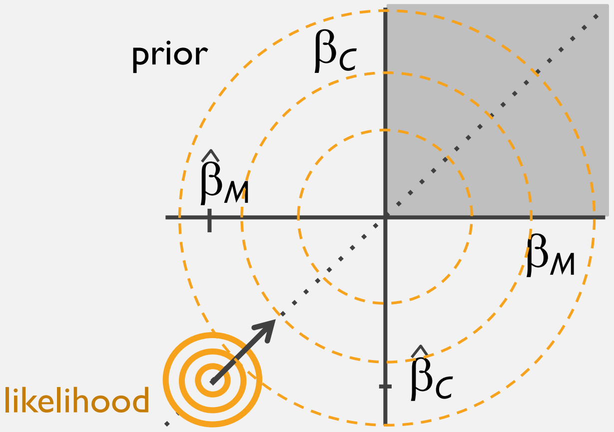
- Output for hypothetical Data A.

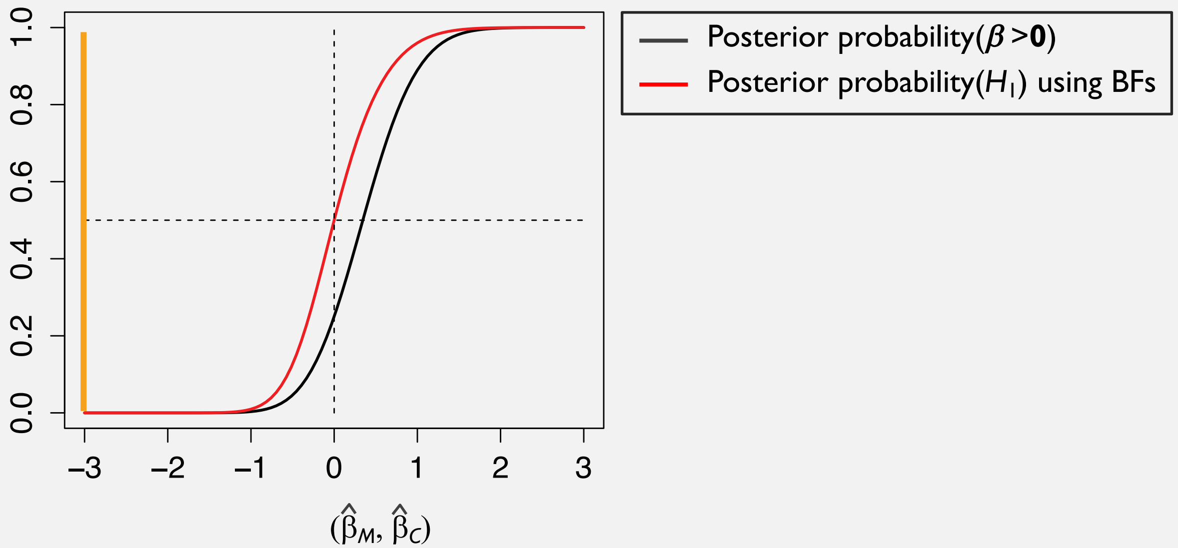
	Data A			
	c_t	f_t	BF(M_t,M_u)	PMP
H_1	1/4	1	4	0.27
H_2	1/2	1	2	0.13
H_3	1/8	1	8	0.53
H_{unc}	1	1	1	0.07



COMPARISON
BAYES FACTOR VS POSTERIOR PROBABILITY

- Consider H_1: $\beta_M > 0, \beta_C > 0$ versus H_2:"not H_1".
- Compare $\Pr(H_1|\text{data})$ with $\Pr(\beta > \mathbf{0}|\text{data})$.





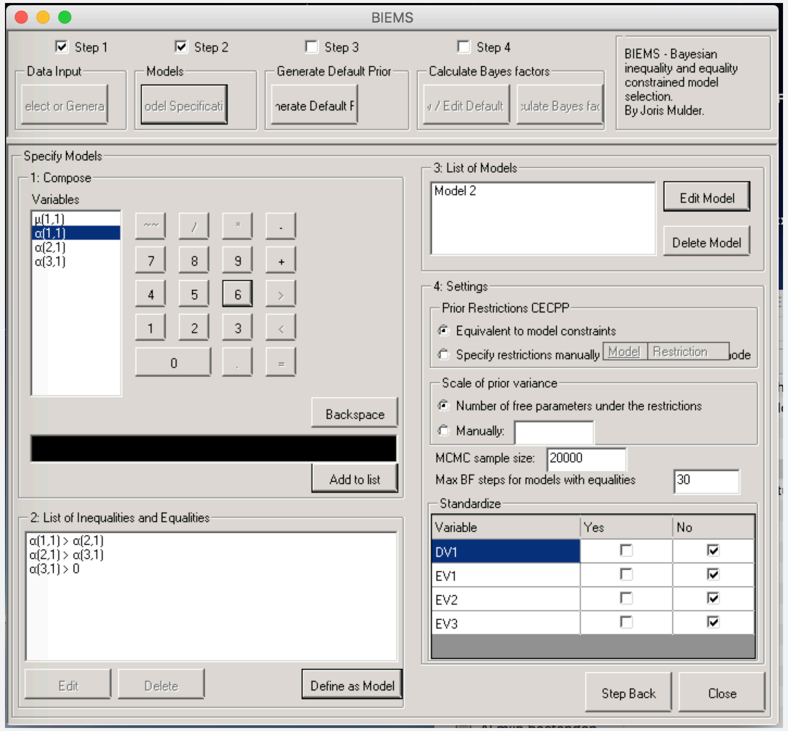
USEFUL PROPERTIES OF BAYES FACTORS

- **Bayes factors**…
  - … have an intuitive interpretation as the **relative evidence in the data** between statistical hypotheses.
  - … can be used for testing **multiple equality and order hypotheses**.
  - … do **not rely on large sample theory**.
  - … do **not rely on the sampling plan**.
  - … are **consistent** in very general scenario's

SOFTWARE FOR BAYES FACTOR TESTING

- **BIEMS** (Mulder et al., 2012) for testing hypotheses with equality and order constraints in the (multivariate) t test, (multivariate) regression, (M)AN(C)OVA, repeated measures .
- BIEMS has a user-friendly interface.
- R-package in development.

SOFTWARE FOR BAYES FACTOR TESTING



SOFTWARE FOR BAYES FACTOR TESTING

- **BOCOR** (Mulder, 2016) and **BCT** (Mulder & Gelissen, in prep.) for testing equality and order constraints on bivariate correlations, partial correlations, polychoric correlations, etc.
- BOCOR and BCT are stand-alone executables.
- R-package in development.

SOFTWARE FOR BAYES FACTOR TESTING

BCT_input.txt

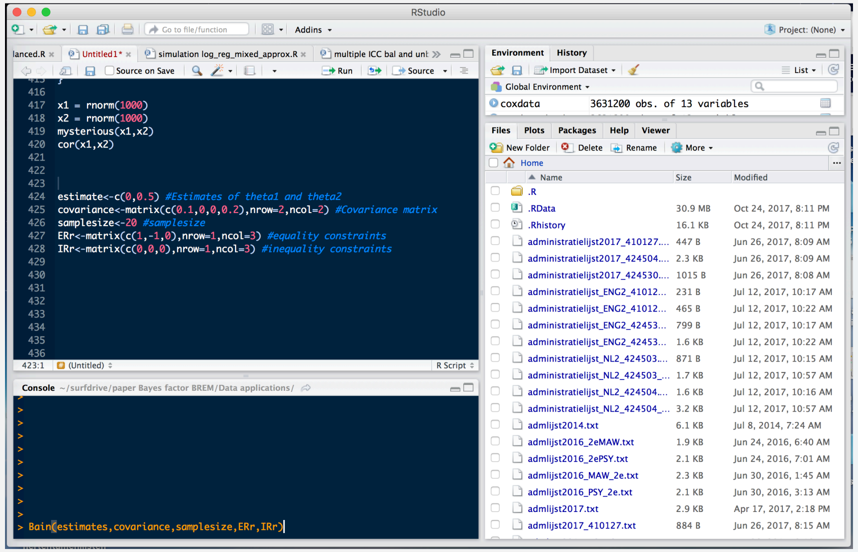
```
Input 1: model
#DV     #covs   intercept      #populations   Ntotal
3       3       1              1              1286

Which DVs are ordinal (0=continuous, 1=ordinal)
1 1 1

Input 2: hypotheses
#hypotheses
2
#equalities, #inequalities
1 1
0 2

Input 3: constraints in hypotheses
Equalities H1; Inequalities H1; Equalities H2; Inequalities H2; etc.
1 2 1 1 3 1
1 3 1 1 3 2

1 2 1 0 -1 0
1 3 2 0 1 0

Input 4: implementation details
iterations, iseed, delta
1000 123 .1
```

BOCOR/BCT!

SOFTWARE FOR BAYES FACTOR TESTING

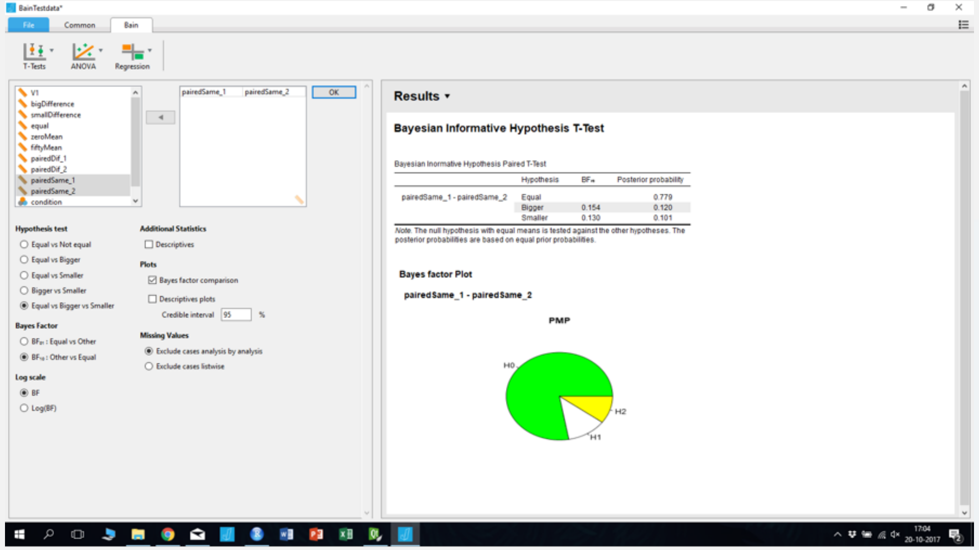
- **Bain** (Gu et al., in press) for testing equality and order constraints on parameters for **general statistical models**.
  - BaIn uses an approximate Bayes procedure that only needs the ML estimates and estimated covariance matrix.
- BaIn is a R-package. It is currently being implemented in JASP (Wagenmakers and colleagues)

SOFTWARE FOR BAYES FACTOR TESTING



Bain in R!

SOFTWARE FOR BAYES FACTOR TESTING



Bain in JASP!

SOFTWARE FOR BAYES FACTOR TESTING

- Other software packages include:
  - **BFvar** (Böing-Messing et al., 2017) for testing equality and order constraints on **group variances**.
  - **BF-ICC** (Mulder & Fox, under review) for testing constraints on **intraclass correlations**.
  - **BF-NAM** (Dittrich et al., in prep.) for testing constraints on **network autocorrelations**.
- Bayes factor testing not for order constraints:
  - **BayesFactor** R-packge (Morey & Rouder, 2015)
  - **JASP** (Wagenmakers and colleagues, 2016).
  - **SPSS** (latest version).

ORDER HYPOTHESES ON RELATIVE EFFECTS REVISITED

- We can combine these order constraints into a single 'order hypothesis'.
  - "All sources of workplace aggression have a positive effect on depression and the effect is largest for managers, followed by coworkers, followed by visitors”
  - Data: N = 864.

Power differential hypothesis: $H : \beta > \beta > \beta > 0$

Analysis in **BIEMS**

BF(H_1,H_u) = 9.1

BF(H_2,H_u) = .46

→ BF(H_1,H_2) = 19.8

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS REVISITED

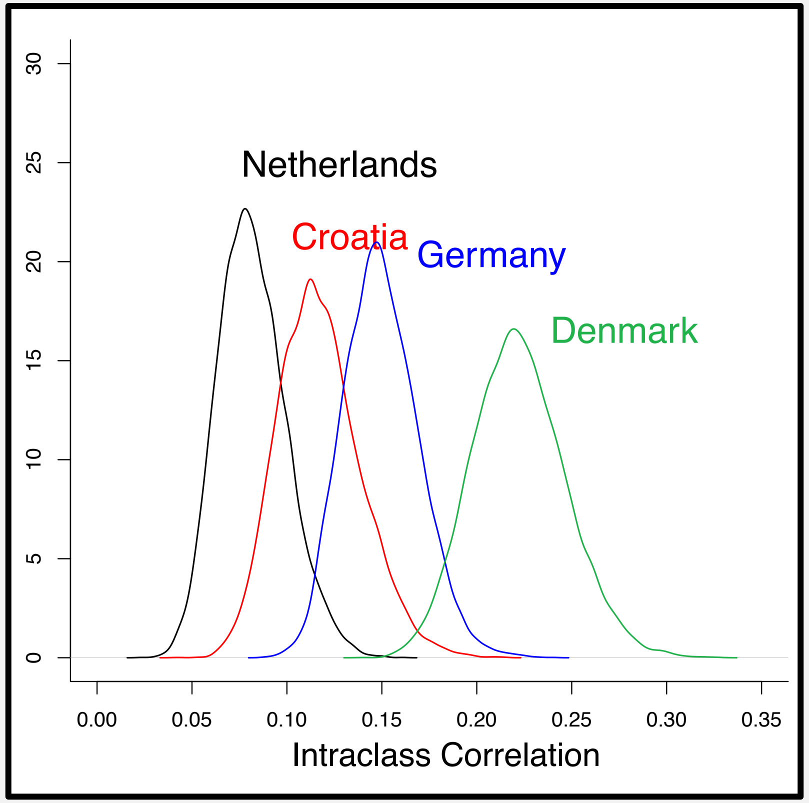
- **Null hypothesis**: H_0: $\rho_{NL} = \rho_{CR} = \rho_{GER} = \rho_{DEN}$.
- **Order hypothesis**: H_1: $\rho_{NL} < \rho_{CR} < \rho_{GER} < \rho_{DEN}$.
- **Complement hypothesis**: H_2: neither H_0, nor H_1.
- Data: 100-150 schools per county; school classes of 15 students.
- **Results:**

Analysis in R (Mulder & Fox, under review)

$BF(H_0,H_u) = .000$	→	$P(H_0\|Data) = .000$
$BF(H_1,H_u) = 18.1$	→	$P(H_1\|Data) = .986$
$BF(H_2,H_u) = .261$	→	$P(H_2\|Data) = .014$

ORDER HYPOTHESES ON INTRACLASS CORRELATIONS REVISITED

- **Posteriors** under unconstrained model



ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS REVISITED

- It is often suggested that information-sharing occurs sooner and at a higher rate among colleagues who they feel related to – this is often attributed to identity.

- H_1: $\beta_{position} > \beta_{building} > \beta_{gender} > 0$
- H_2: $\beta_{position} > \beta_{building} = \beta_{gender} = 0$
- H_3: neither H_1, nor H_2.

→

$P(H_1|Data) = .06$

$P(H_2|Data) = .94$

$P(H_3|Data) = .00.$

ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS REVISITED

- It is often suggested that information-sharing occurs sooner and at a higher rate among colleagues who they feel related to – this is often attributed to identity.
- Data: 1505 emails among 51 colleagues.

- H_1: $\beta_{position} > \beta_{building} > \beta_{gender} > 0$
- H_2: $\beta_{position} > \beta_{building} = \beta_{gender} = 0$
- H_3: neither H_1, nor H_2.

→

$P(H_1|Data) = .06$

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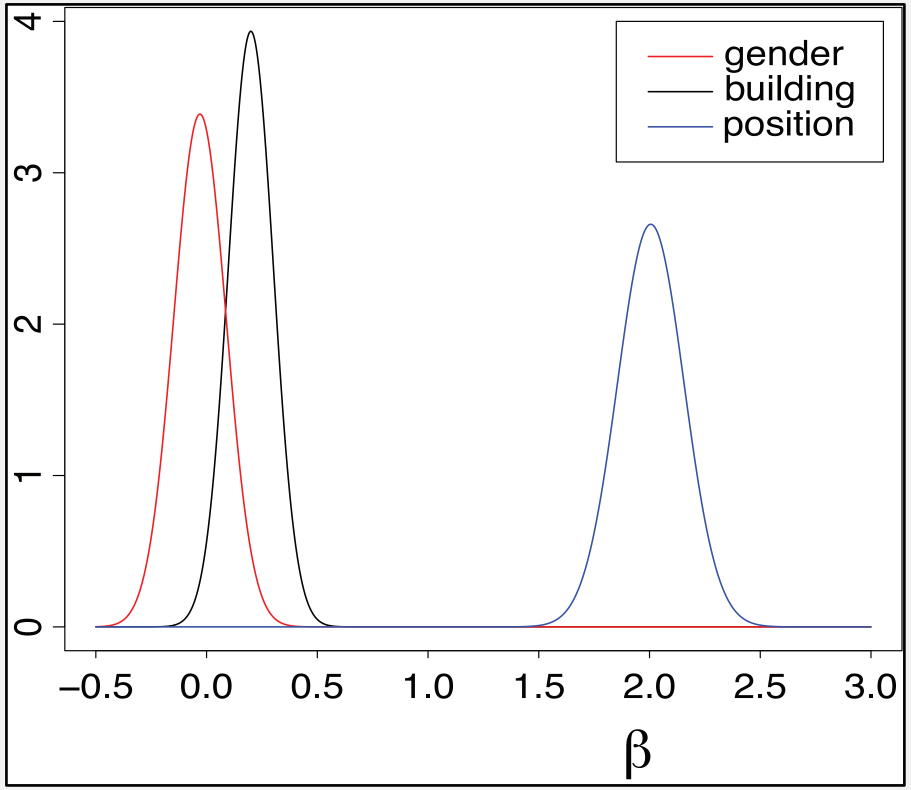
ORDER HYPOTHESES IN DYNAMIC SOCIAL NETWORKS REVISITED

- **Posteriors** under unconstrained model



CONCLUSIONS

- The Bayes factor has many useful properties for testing statistical hypotheses such as
  - its **intuitive interpretation** as the statistical evidence between competing hypotheses;
  - the ability to test **multiple hypotheses** with **equality and order constraints**;
  - its **consistent** behavior or their invariance for the **sampling plan**.
- **Software** for computing Bayes factors is becoming more and more available **for challenging testing problems**.

THANK YOU!

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